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A NEW THEORY OF THE SURFACE MARKINGS OF THE MOON.

E. MILLER.*

FOR POPULAR ASTRONOMY.

Our next door neighbor, the Moon, although so near, has its mysteries. The problems arising from the lunar theories are so many and of such complex nature, that as yet no complete solution of them has been reached. The dominant features are the craters, and it is practically true that all lunar theories begin with them. The mountain ranges, the craters, the cracks and seams upon the Moon, are clearly and distinctly defined with the assistance of a good telescope; and by means of the art of the photographer, there are today better maps of the lunar surface that is exposed to our view, than any maps ever constructed of a corresponding portion of the Earth's surface.

Three theories have attracted considerable attention, to which a large amount of industry, patience and enthusiasm, has been given. If not considered presumptious, I shall add another theory, one that is entirely new and original, so far as is known. A brief outline of the first, second and third will be given before touching upon the fourth. The first is known as the "volcanic theory," the second as the "meteoric," and the third as the "tidal theory."

All known active and extinct terrestrial craters are volcanic beyond a doubt, and the great mass of writers affirm the same to be true of the lunar craters, all of which that are exposed to our view being, however, of the extinct variety. No craters upon our globe begin to approximate in size the largest of the Moon. There are lunar craters 800 miles in diameter, and they range from that size down until they become imperceptible. Those large plains upon the lunar surface, heretofore called seas and of oval outline, are now by some classed as craters. The largest known crater upon Earth does not exceed seventeen miles in diameter. Volcanic action upon our neighbor, the Moon, must have been in its day much more teriffic than anything of a

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similar nature upon our own planet. The number of lunar craters visible to us is somewhere in the neighborhood of 30,000. Estimating one-half of the Moon's surface at about 7,000,000 of square miles, and the surface of North America, excluding Greenland and the West India Islands, at 8,000,000, we shall find by the most reliable statistics that North America has only one-tenth as many craters, both active and extinct, as one-half the Moon's total surface. This apparent discrepancy may be accounted for when it is considered that upon the Earth two powerful agencies—aqueous and atmospheric—have been at work for perhaps millions of years, producing a demolition of craters, or burying them under successive layers of geologic *debris*. Without discussing the "volcanic theory" in detail, this simple reference to it must be considered sufficient. Mr. G. K. Gilbert delivered a very able and interesting address, Dec. 10, 1892, before the Philosophical Society of Washington, on "The Moon's Face," of which the following may be considered the main thought. Basing his discussion upon the supposition that when the Moon was in the form of a ring similar to the rings of Saturn the time came when the stability of the lunar ring was destroyed and an indefinite number of moonlets, constituting in fact, a meteoric swarm, resulted. Under the superior attraction of one or more of the largest masses, a trend towards that one was set up, and the impacts or collisions thus produced not only fused the striking meteorite but also that portion of the nucleus that was struck. A coalescence took place, the central body grew in size, and its power of attraction increased accordingly. In harmony with the laws of celestial dynamics, the velocity of the moonlets towards the central or controlling mass would increase, and the energy expended would be sufficient to melt every moonlet, even though it should be composed of the most refractory substance. The portion of the surface of the central mass struck by the moonlet would itself be fused, and there would be formed a cup shaped depression similar to the crater forms now seen upon the Moon, together with their attending phenomena. The larger the moonlet, the larger and deeper would be the depression and the greater the fusion of the rock material. The smaller the moonlet, the shallower would be the crater's depth, and less its diameter. I have seen in the cuts and gullies made by the overflow, and backwater, and subsequent subsidence, of the San Pedro river of Southern Arizona, the plastic mud deposited by the falling waters of such consistency, that when a pound or a two-pound rock was dropped into that mud, an examination of it

the next day showed a crater-like form resembling exactly the forms of the lunar craters, taking the normal type as an example.

"If a drop of water be made to fall on a still surface of water, the outward moving annular wave at one instant encloses a crater; at the next instant a mouud rises in the center of the crater." In the case of the viscous mud, as shown in the example of the San Pedro, if a stone fall from a certain height, "a cup-like cavity or crater with a smooth rim will be produced; if it fall from a greater height it will produce a larger cup with smooth rim and with a smooth dome-like hill in the center." An experiment of this sort can easily be performed. The white streaks, constituting one of the great lunar features, is by this theory of impact also accounted for. According to Dr. B. A. Gould, "The most remarkable appearance on the Moon, for which nothing on Earth furnishes an example, is presented by those immense radiations from a few of the larger craters—perfectly straight lines, as though marked with chalk along a ruler—starting from the center of the crater and extending to great distances over every obstruction. My explanation is that a meteorite, striking the Moon with great force, spattered some whittish matter in various directions. Since gravitation is much feebler on the Moon than with us and atmospheric obstruction of consequence does not exist, the great distance to which the matter flew is easily accounted for." Accepting the premises upon which the "meteoric theory" is based as true, all the characteristics of the Moon's surface—craters, streaks, central domes, rising from the floors of the craters, and even the building of the Moon itself,—can be explained on the theory of the impact of moonlets or meteorites upon the central mass.

But it may be stated that the impact of the larger masses upon the central one, in order to produce results such as we see on the Moon's surface, is a theory that contains inherent and radical difficulties. The diameter of the Moon is about 2100 miles, and by this theory we have a moonlet whose impact was capable of producing a crater of 800 miles in diameter. We may reasonably suppose that when the lunar ring was broken into fragments, the central mass may not have had a diameter of over 1,000 miles, and collisions would take place not long after the dismemberment. A collision between two bodies, one 1,000 miles in diameter and the other 800 miles in diameter, would break both bodies into a multitude of smaller bodies, and hurl them outward into space, in a thousand different directions. This process would be carried on just as fast as the meteoritic masses of the old ring

should be drawn towards the center. A swarm of asteroids, would therefore, under such conditions, have taken the place now occupied by the Moon. Yet it is fair to assume that the "meteoritic theory" is true as one of the factors in any lunar theory that may be advanced. The "volcanic" is also another factor that must be considered as equally important. There is therefore no objection to the statement that both forms of action, the volcanic and the impact of meteors, may have had, each its due share in bringing about the present condition of the Moon's surface.

The third or "tidal theory," supposes a time when the Moon was liquid but covered with a thin crust. "The Moon then rotated more rapidly than now, and great tides, excited by the Earth's attraction, rocked and cracked its crust, and here and there squeezed out a portion of the liquid nucleus, which flowed back again when the tidal wave had passed; but congelation caught the flood at its edges, so as to mark its limit by a solid ridge. By each successive tide the operation was repeated, with the result that the wall was given a circular form and was gradually built up. The process was finally closed by the congelation of lava in the orifice, and while congelation was in progress the last feeble eruption sometimes produced a central hill." The "tidal theory" is quite strong in certain directions, while in others it is weak.

I now desire to pass by these three forms or theories, and construct another which may or may not have merit of its own, a system of lunar dynamics, which so far as I know has never been looked at in the light in which I propose to consider it. The suggestion is pertinent right here, that this fourth theory does not leave out of sight the other three as factors of importance in the solution of the problem in hand.

After the Moon had collected her material together into the form of a globular mass, and before becoming a rigid body as it is now known to be, that is when still a gaseous or liquid body, there is scarcely a doubt as to the existence of a force within the Moon powerful enough to produce each and all of the peculiar markings of the lunar surface. The most important member of the solar system, the Sun, is in either a gaseous or liquid state and has been so for millions of years, and will continue to be so for millions of years more. During all these years the Sun has been, and will continue to be, diminishing in size at the rate of about 250 feet in diameter per year. The annual contraction of the Sun's mass will in time not only solidify the Sun, but dimin-

ish very rapidly toward the end the amount of heat given off. Finally the condition of the Sun may become that of a cold, dark body, destitute of both heat and light, and if there be no atmosphere and aqueous vapor such as we are acquainted with, then the solar surface will present an appearance similar to that of the Moon—a surface covered with mountain ranges and peaks, craters, fissures and streaks. But if an atmosphere and aqueous vapor should be in existence at that remote time of the future, then surface markings resembling those of the Earth will characterize the surface of the Sun.

The contraction of the Sun at the rate of 250 feet per annum is sufficient, according to the theory of celestial dynamics to produce all the heat which the Sun now gives off, and moreover is enough for that purpose until the Sun is diminished to the size of a body sensibly smaller than now. This contraction of the Sun and consequent radiation of heat may in part be the form of energy, or one of the forms of energy, in operation upon and within the Sun, to which sun-spots owe their origin. Sun-spots range in diameter from 500 miles to 100,000 miles or even more, and the penumbra is still greater. As to their depth, they seldom exceed 2500 miles, although in one instance, observed by Mr. Langley, the enormous depth of 5,000 miles was probably reached.

The force, or forces, at work upon and within the Sun, will after the lapse of ages, leave the body of the Sun in a viscous condition, and spots formed under such circumstances will become permanently fixed in the form of craters possessing the essential characteristics of the lunar craters. Not only the forms that are tppical or normal but those that are broken and misshapen, with rims, in some cases elevated thousands of feet above the surrounding surface, but slightly pushed up in others; with cavities reaching in some instances to immense depths; and with fissures and streaks, in straight or irregular lines, will all be found to dot the solar surface as abundantly as do those upon the face of the Moon. If there should be neither atmosphere nor moisture, then and there, all the rugged forms and appearances, the precise counterparts of of those upon the Moon, will mark the period when the Sun will have ceased to radiate heat and light throughout the solar system, and will have become a dark, dead body ready for some great castrophe by which it may be transformed into another and perhaps more beautiful system.

Taking the Sun as an example of a mode of action, is it beyond

the realm of truth and of science to formulate a theory for the Moon similar to that which is now at work, and is actually passing before our eyes, upon the Sun? The nebular hypothesis gives us a starting point—a lunar ring, at first complete it may be, then broken into a multitude of fragments. Gravitation molded these fragments into a veritable planet that was to be our nearest neighbor. In its earliest stages, the Moon was not a solid, rigid body, but a mass of gaseous or liquid matter in a condition of white heat. In time it contracted its volume, heat was evolved, and intense action set up within the mass. The results would be noticeable upon its surface, and manifestations of protuberances and other phenomena similar to those of the Sun, would naturally follow. As the ages rolled by, the heat would disappear in part by radiation, and in part by absorption, to be stored away as latent heat. As there was no atmosphere nor moisture, the resulting form of the Moon's surface then would constitute a photograph of its condition true to life as long as the solar system should endure.

In conclusion, therefore, is not the assumption that the moon-spot theory, to coin a term, is in sufficient harmony with well established mathematical and dynamic principles to constitute a theory of as great importance as any other, and perhaps more satisfactory in that it meets the conditions of astronomical science at all points?

This fourth theory does not claim to discard entirely volcanic agencies, or the impact of meteorites, or tidal action, but it does claim to be more general in its operations, more wide-reaching, and yet allowing to a certain extent the agency of other forces as restricted and collateral, within limits.

The contraction of the Sun's mass, the development of heat, and the immense energy consequent upon that development, as seen in the splendid outbursts of the protuberances, the formation of sun-spots which crossing the Sun's disc, present such wonderful changes in form, size and movement, may not all this be but a transcript of the story that the Moon might tell concerning the events that took place upon and within the lunar mass millions of years ago? May not the present condition of the Moon's surface be a prophecy of what the Sun's will be in the far distant future?

The energy of lunar heat when the Moon was an incandescent body must have manifested itself in results similar to those which the energy of solar heat now produces. The one is but a type of the other.

COMET *c* 1895. (PERRINE).

C. D. PERRINE.

FOR POPULAR ASTRONOMY.

This comet has now passed its perihelion and has made its closest approach to the Earth. It is now an evening object but is again approaching conjunction with the Sun which occurs on January 11th when it again becomes a morning object. It is rapidly decreasing in brightness and owing to this fact and to its unfavorable location has not been seen here recently. In fact I have seen it but once since it passed the Sun on December 16, and that was on December 20 when with the 12-inch telescope it was visible for a few moments before it sank below the horizon. Unfavorable weather about this time doubtless prevented its being seen more than the once.

The comet has been photographed on all possible occasions by A. L. Colton and a good series of negatives obtained. Cloudy weather interfered when the comet was rapidly growing brighter and just before it became wholly immersed in the morning twilight. This was unfortunate as about this time the comet showed signs of increasing activity. A photograph taken on December 8 shows a number of flutings in the main tail with indications of the condensations noticeable in the photographs of some of the recent bright comets. Several short streamers had also appeared in the neighborhood of the two principal tails, projecting backward from the head.

The first photograph secured was on the morning following discovery. This photograph as well as all subsequent ones shows two tails. The shorter of the two is intrinsically the brighter and is situated about 30° to the north of the other. This angle between the two tails does not remain constant, however, but is subject to fluctuations.

The illustration* accompaning is from the negative taken on November 26th. It is best suited to reproduction and has been enlarged $1\frac{8}{10}$ diameters from the original. On this plate the longer tail can be traced for a distance of $3\frac{1}{2}°$. On the later negatives the tail can be traced still further.

From observations by the writer on November 18, 25 and December 1, Professor Campbell obtained the following elements:

$T = 1895$, Dec. $18^d\,32570$ Greenwich M. T.
$\omega = 272°\ 35'\ 38''.0$
$\Omega = 320\ \ 26\ \ 19\ .1$ } 1895.0
$i = 141\ \ 39\ \ 22\ .6$
$q = 0.192253.$

* See frontispiece.

Residuals (O — C) : cos $\beta' \Delta\lambda' = -1''.9$; $\Delta\beta' = -2''.1$.

The agreement of the first set of elements with the above is very satifactory.

LICK OBSERVATORY, Jan. 2, 1896.

PLANETARY PHOTOGRAPHY WITH A REFLECTING TELESCOPE.

J. M. SCHAEBERLE.

FOR POPULAR ASTRONOMY.

The astronomical telescopes in general use at the present time are practically of the same design as those of four score years ago. In the case of the refractor the power has, indeed, been largely augmented but almost wholly by the mere brute force of increasing its size, accompanied at the same time by an increase in cost so enormous that to overtop the giant refractor of the present day the wealth of a nation or of a Rothschild must be appealed to.

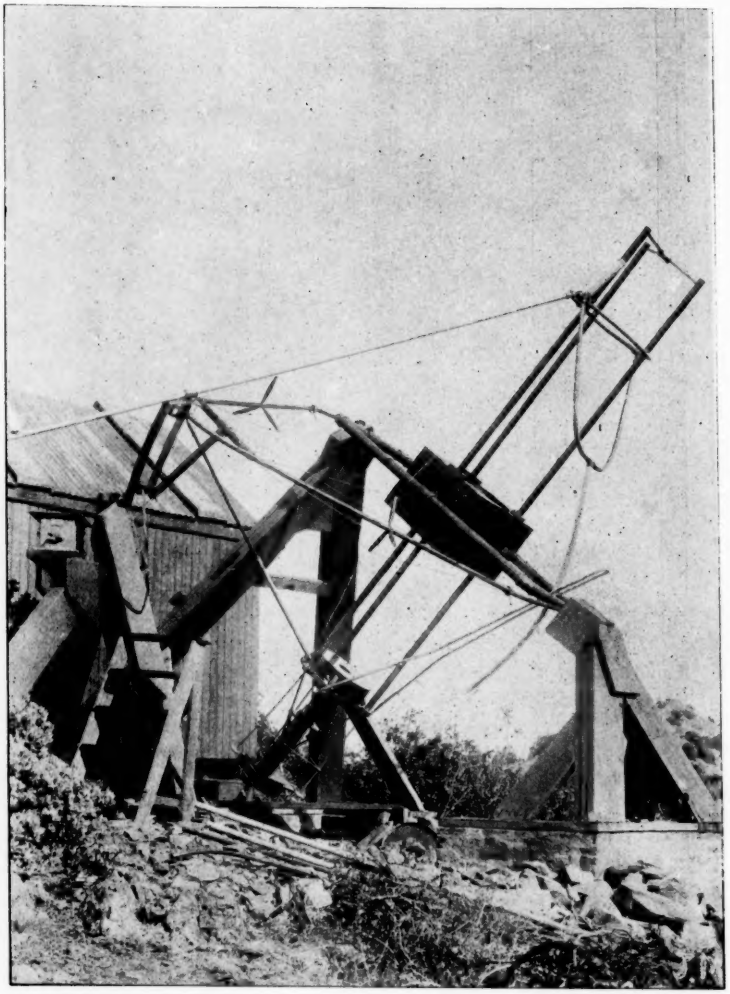
In spite of the acknowledged fact that in practice the refracting telescope is, as a rule, visually superior to the reflector the former is and must (so far as our present knowledge goes) continue to remain an imperfect instrument.

From a considerable experience with both forms of telescopes it seems to me that there are only two reasons why the refractor as a rule surpasses the reflector in visual observations.

The first and main reason is that the reflecting surface used may not be as well figured as man can make it; and secondly the peculiarities of the ordinary form of mounting reflectors introduces abnormal, and ever present diffraction phenomena caused by the secondary mirror, and its supports, coming between the celestial object and the reflecting surface, thereby destroying the continuity of the light wave front.

The want of a proper figure is not necessarily the fault of the optician, it may be, and often is due to varying internal stress in the material of an originally perfect mirror, caused by changes of temperature, and by alteration in the position of the mirror with reference to the vertical. For the latter of these reasons it would seem that so long as we are obliged to use material which suffers appreciable changes of form under the action of gravity, the best results should, as a rule, be expected from moderate size mirrors.

PLATE X.



18 Inch Reflecting Telescope for Planetary Photography. Equivalent Focal Length 330 Feet. Mounted at the Lick Observatory.

Popular Astronomy, No. 26.

The diffraction effects peculiar to the ordinary form of mounting can be completely overcome, if desired, by using an eccentric area (circular) of the parabolic mirror; however only about one-fifth of the total area of the mirror can then be utilized.

As the only known method of causing a system of originally parallel rays of heterogeneous wave-lengths to pass through a given point is by reflection from a properly figured surface, the reflecting telescope has, theoretically, everything in its favor, and especially so for photographic purposes.

The excellence of the photographs which have been obtained with the refractor is almost wholly due to the fact that the plates used are most sensitive to light rays of a particular wave-length, and so long as there is no over-exposure, only these particular rays register their presence on the photographic plate. When the emulsion is discovered which is not selective between such narrow limits the reflector will be the ideal and only instrument for photographic purposes which can utilize such an improvement in the sensitive plate.

Judging from some experimental photographic results obtained at the Lick Observatory with the instrument shown in Plate X* it seems certain that with a moderater size parabolic reflector and a good secondary hyperbolic mirror (both rigidly mounted in a skeleton tube not more than ten or fifteen feet long and protected from the wind) as good results may be expected from the photography of the planets and other bright celestial areas of limited extent as can be obtained with the largest and most powerful photographic refraction of the present day.

To construct an instrument which will give such results does not require much monetary outlay if the observer is willing to devote (for months if necessary) those hours ordinarily given to recreations, to the work of figuring the mirrors. He should have an inexhaustable supply of patience, and confidence in his own ability to do the work.

Three main requisites must be strictly fulfilled:

1st. The large mirror must be a portion of a paraboloid of revolution of so perfect a form that no test however severe will reveal reflecting areas which could, by further polishing, be improved. A few scratches need not discourage the beginner, as a properly figured surface with many scratches will give excellent results where an approximately figured, highly polished mirror

* For a description of this telescope see "Publications of the Astronomical Society of the Pacific," Nos 42 and 44. Also the October, 1895, number of the "Photographic Times."

without a single scratch will be useless for the present purpose, although fair results might be obtained in the principal focal plane. If the figuring of the large mirror is not as perfect as man can make it, it will be a mere waste of labor to construct the remaining parts of such an instrument as we are here considering.

2d. The secondary reflector must be either the concave surface of an ellipsoid of revolution or the convex surface of an hyperboloid. My own experiments with secondaries have been confined almost exclusively to the *plane* and *convex* forms.

The plane surface can be tested in a number of different ways which need not be considered here as the results obtained with a Newtonian reflector are all confined to a scale of the image corresponding to the focal length of the large mirror, and for planetary photography this scale will be much too small.

The tests for figuring the concave mirror must all be made in the telescope with the two reflectors properly adjusted. All the constant imperfections of the image in the secondary (new) focus will then be due to the faulty surface of the small mirror, if the large mirror is known to be parabolic.

3rd. The mountirg must be such that the image to be photographed can be made to remain perfectly stationary on the photographic plate. For securing this end the short tube is evidently of the greatest advantage. If the sector-arc is made of large radius the effects of the irregularities in the running of the driving clock acting on this sector can be very much diminished.

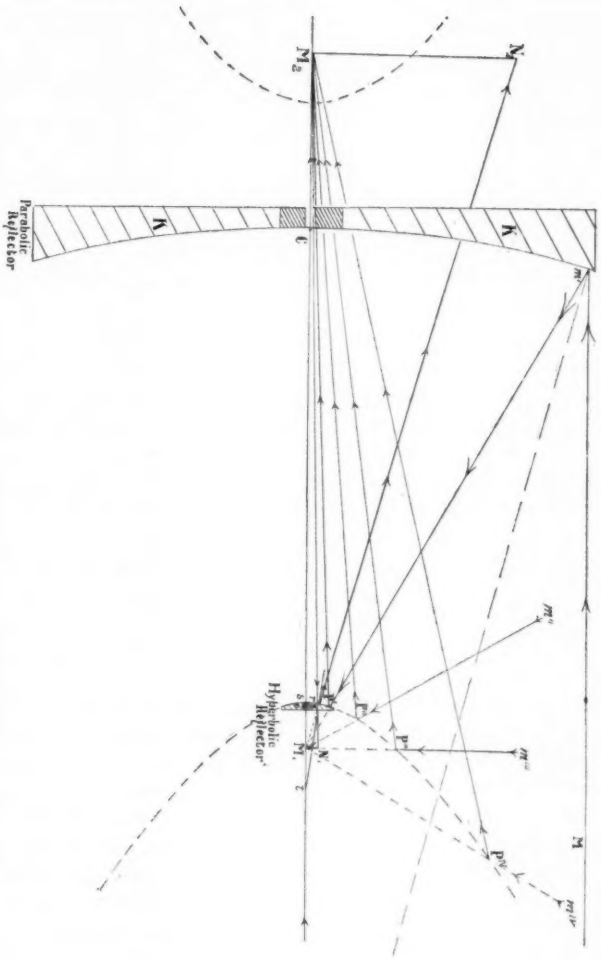
By a simple system of sliding rods and clamps, all the moving parts can be made to brace each other, thus giving great rigidity to the telescope. The photograph, Plate X, will give a fair idea of how this is done in the case of the Lick Observatory reflector.

The great value of this form of telescope for photographic purposes is its adaptability to any scale.

A planetary image of any desired size (always greater than that given by the parabolic reflector alone) can be made to form at any desired point on the optical axis by simply giving the secondary mirror the required surface curvature.

The readers of POPULAR ASTRONOMY will most readily understand the optical principles involved by referring to the diagram given opposite.

All rays parallel to the optical axis of the parabolic mirror K and reflected from the latter are directed to the focus M_1. If, however, before reaching this focus they are reflected from a



second mirror, B, in such a way that all the rays now pass through a second point, M_2, on the axis, it is required to prove that the form of the secondary convex mirror must be that of an hyperboloid of revolution.

Let it be supposed that the parabolic mirror is of indefinite extent and that the lines $m^{IV}\ P^{IV}$, $m^{III}\ P^{III}$, $m^{II}\ P^{II}$, $m^{I}\ P^{I}$, etc., are portions of the light rays after reflection from the parabolic

mirror. Now to fulfill the required conditions the inclination of the reflecting surface at the several points of incidence must evidently be such that all the angles $M_1 P^{IV} M_2$, $M_1 P^{III} M_2$, etc., are bisected by the curve at the several points P^{IV}, P^{III}, P^{II}, etc. It is a well known property of both the ellipse and hyperbola that lines drawn from the two foci (of either curve) to any point of the curve make equal angles with a tangent to the curve at the given point. It at once follows that in the present case the curve which passes through the points P^{I}, P^{II}, P^{III}, P^{IV}, etc., is an hyperbola the foci being at M_1 and M_2; for by construction lines drawn from these two points to the several points on the curve make equal angles with the tangents at the several points. As M_1 is the focus of both the hyperbolic and parabolic mirrors it follows that *to secure perfect definition the nearer focus of the hyperbolic reflector must coincide with the focus of the parabolic mirror, and the image formed must be somewhere on the common optical axis of the two reflectors.*

Ordinarily the image is made to form nearly in the plane of the large mirror (at the point C in the diagram) but for photographic purposes decided advantages are gained if the sensitive plate can be placed at some distance (H) back of the large mirror for not only is the sky area to which the plate is exposed, reduced thereby, but the size of the plate which can be completely covered with all the rays coming from the secondary increases with H ($= CM_2$).

An image having the linear dimension $M_1 N_1$ in the focus of the large mirror will have the dimension $M_2 N_2$ in the plane of the photographic plate, (the given combination of the two mirrors is here made the same as that actually existing in the telescope mounted at the Lick Observatory). The more heavily shaded areas of the two mirrors K and B illustrate the actual angular size of the reflecting apertures; and only that portion of the image $M_2 N_2$ can be photographed which receives the rays passing through the opening in the center of the large mirror.

The fundamental formulæ which I have deduced for reflecting telescopes in general are given below. As this paper would be more technical in character than might be desired, I have thought it best to give simply the formulæ to be used in practice. Those readers of "Popular Astronomy" who wish to follow out the methods of derivation of the formulæ are referred to a paper of mine on this subject to be published in a forthcoming number of the *Astronomical Journal.*

Let $F = M_1C$ denote the focal length of the parabolic reflector.
Let $r = ts$ denote the radius of curvature of the small mirror (the focal distance of this mirror is sM_1).
Let $H = M_2C$ denote the distance of the photographic plate from the center of the parabolic reflecting surface.
Let L denote the equivalent focal length of the telescope so that

$$\frac{L}{F} = \frac{M_2N_2}{M_1N_1}$$

Then the required expressions are:

$$L = \frac{F}{r}\left[(F + H) \pm \sqrt{r^2 + (F + H)^2}\right]$$

$$r = \frac{2 \,.\, L \,.\, F\,(F + H)}{L^2 - F^2}$$

$$H = \frac{r\,(L^2 - F^2)}{2 \,.\, L \,.\, F} - F$$

The uses of these equations are as follows:

1st. If both mirrors are already finished and their constants measured, the equivalent focal length L for any value of H can at once be found by means of the first equation. The adopted value of H should always correspond to that point at which the eye is placed for testing during the process of figuring the small mirror —in other words, at the point where the definition is best.

2d. If the large mirror is finished and it is desired to have a given equivalent focal length L for any convenient value of H the second equation will give the radius of curvature to which the secondary must be ground. All the testing during the progress of figuring should then be made at the distance H from the large mirror.

3rd. If both mirrors are finished and it is desired to have (within certain limits on each side of H) a particular scale for the image, the third equation will, for a given value of L, give the distance H from the large mirror at which the photographic plate must be placed to produce the desired scale.

The formulæ refer to the theoretical condition of perfect definition. In practice the *angular* area of the reflector is so small, that the sphere, the ellipsoid of any eccentricity, the paraboloid, and the hyperboloid of any eccentricity will for the same value of *r*, be so nearly coincident with the surface of any secondary reflector having the same radius of curvature in the optical axis, that all of these surfaces will give moderately well defined images at *any* point on the optical axis; so that the image can be made

to vary in size by simply shifting the secondary along the axis of the telescope, thereby causing more rapid changes in the values of H and L.

Each change of position really assumes a change in the eccentricity of the secondary, but since this element is fixed for a given mirror there is only one value of H (and L) for a given secondary and this value fixes the point at which perfect definition is secured.

Applying these formulæ to the case of 18-inch reflector I first found by actual measurement

$$F = 12.304 \text{ feet.}$$
$$r = 1.188 \text{ feet.}$$

By means of a sliding tube the distance of the plate can be varied between the limits H = 3 feet and H = 4 feet; the range of L is, therefore, according to the first equation

For H = 3 feet.....L = 321.2 feet
" H = 4 feet.....L = 342.0 feet

From which it appears that L here increases more than twenty times as fast as H. The rate of increase for any other values of F, *r* and H can be found from the expression

$$\frac{\Delta L}{\Delta H} = \text{Rate} = \frac{2 \cdot L \cdot F^2}{r(L^2 + F^2)}$$

For photographing the Sun or the Moon the secondary mirror must be backed by a concentric circular disk of such a diameter that all *direct* light from either of these bodies is cut off from the sensitive plate.

For photographing the Moon and all the major planets except Venus and Mercury (for which the exposures are very short) it is necessary to give exposure times varying all the way from a large fractional part of a minute up to several minutes; it is therefore necessary to make use of a guiding eyepiece in addition to a driving clock. The one I employ is fastened to the plate-holder and has a focal length $f' = 2$ inches the magnifying powers $(= \frac{L}{f'})$ for H = 3 feet and H = 4 feet are, therefore, respectively 1927 and 2052.

Although the guiding objects are ordinarily much over magnified, still for the purpose of accurate following these high powers are of great advantage.

I devised many different methods for keeping the image stationary on the plate but all of these special arrangements were more

or less unsatisfactory. I now use a most primitive and completely satisfactory method for correcting all the small displacements of the image (due to variations in the running of the clock, changes of flexure in the tube and shifting of the mirrors). A very gentle pressure against the lower extremity of the instrument is the simple device. The hand is steadied by a stationary support placed close to the tube but wholly free from the same.

In this way an exposure of five, ten or more minutes can, if required, be made with the greatest ease and accuracy. The only movements of the image which I have not as yet been able to control completely are the rapid, vibratory ones caused by the wind, to which the whole instrument is freely exposed.

LICK OBSERVATORY, University of California,
December, 1895.

MICROMETRIC MEASURES OF THE DIAMETERS OF MARS GIVING BESIDE THE SIZE AND THE ELLIPTICITY OF THE PLANET APPARENT EVIDENCE OF A TWILIGHT ARC.

PERCIVAL LOWELL.

FOR POPULAR ASTRONOMY.

During the summer and autumn, 1894, 464 micrometric measures of the diameter of Mars were made at this observatory.

NUMBER OF MEASURES OF THE DIAMETERS OF MARS.

Equatorial Diameters	256
Polar Diameters	208
	464

Of these	403	were made by A. E. Douglass.
	50	were made by Professor W. H. Pickering.
	11	were made by Percival Lowell.
	464	

The 403 taken by Mr. Douglass, in addition to their general value, these prove to be of peculiar interest. For on reducing them I find that beside furnishing, from their great number, relatively accurate values of the equatorial and polar diameters and of the polar flattening, they yield a by product as unexpected as it is important. Their discussion reveals what appears to be unmistakeable evidence of a twilight upon the planet, sufficiently pronounced to be visible from the Earth and actually to have been measured, unconsciously. That Mars possessed an atmosphere we had what amounted to proof positive before; but that the fact should again be brought to light in this literal

manner, as a silver lining to a cloud of figures is a point of some curiosity. For the measures had no such end in view; indeed, to detect the presence of an atmosphere by measures of the diameters had not suggested itself to any one. Yet, as will be seen, the quantities upon which the evidence rests are so large as to be quite without the pale of accidental error, being ten times as large as the probable errors of observation, and twice as large as the differences that disclose the polar flattening. That up to this time they should have escaped detection is due to their having been masked by another factor affecting the size of the polar diameter, the varying irradiation from the polar cap. To the hitherto unsuspected presence of the first of these factors and to the difficulty of duly evaluating the second are attributable in all probability the discrepancies in the values previously found for the ellipticity.

The first of Mr. Douglass' measures were made on the 6th of July and the last on the 21st of November, 1894. From the 12th of October the planet was measured by him nearly every other night; and in the following manner: the longitudinal thread of the micrometer having been set to the planet's phase axis or polar axis as the case might be, the field of the micrometer was so adjusted that the stationary transverse thread lay close to, but not on, the limb of the planet's disk. The movable thread was then carried some way beyond the other limb and steadily brought back till it lay to the same side of, and at the same distance from the other limb, that the stationary thread lay from the first one. It was then read. During the process of bringing it up its motion was never reversed. In case of overrunning it was carried some distance back and again continuously brought up. Thus any possible backlash was prevented from affecting the measures.

The whole field was next shifted till the stationary thread occupied the same general position toward the second limb that it had done toward the first, and the moveable thread was brought past it till it reached a corresponding distance from the other limb, upon which it was read. The whole field was then shifted to the first position and the process repeated. Each reading thus furnished half the data to two measures.

In the measures made during September, October, and November, Mr. Douglass took the further precaution to place himself so that the line joining his eyes was kept either perpendicular or parallel to the diameter measured; a record of his position being entered opposite the measures. Such record appears in column

4 of the table: E. S. standing for "eyes parallel to single thread," that is, parallel to the longitudinal thread of the micrometer; and E. D., "eyes parallel to double (i. e. transverse) threads." E. S. therefore denotes a horizontal measurement; E. D. a vertical one. As measures were taken in both positions for each diameter at various times, we have here a comparison of some value as will appear further on.

To eliminate systematic errors as much as possible a circular disk was constructed, carefully measured, and carried to the top of Mt. Agassiz, one of the San Francisco peaks, where it was placed just under the summit, facing the Observatory. From triangulation the angles by which its position deviated from the vertical in the first place and from parallelism to the dome in the second were found to be respectively: 6° and 2° west; its distance from the dome in an air-line being 45,330 ft., whence its apparent diameters were:

vertical diameter	:	21″.08
horizontal diameter	:	21 .22

The size of the artificial disk was thus such as to subtend at that distance from the Observatory an angle very nearly equal to the planet's disk. The diameters were then observed through the telescope and micrometrically measured by Mr. Douglass.

Of the measures of the planet, the first to be reduced were those taken from Oct. 12 to Nov. 22 inclusive. They are as follows:

TABLE I.

POLAR DIAMETERS.

Measured by Mr. A. E. Douglass.

Time.			No. of Meas.	Eyepiece		Wt.	Uncor. Mens.	Cor. for Refrac.	Cor. for Irrad. on Limb.	Cor. for Irrad. on Term.	Cor. for Phase.	Reduced to Distance Unity.
	h	m					″	″	″	″	″	″
Oct. 12	13	45	5	862		5	21.95	21.95	21.85	21 79	21.81	9.40
15	13	42	6	"	E. R. A.	4	21.84	21.84	21.74	21.67	21.68	9.36
17	15		4	"	E. S.	4	21.77	21.77	21.67	21.59	21.60	9.38
19	15	5	5	"	E. S.	2	21.92	21.92	21.82	21.73	21.73	9.46
20	13	57	5	"	E. S.	5	21.69	21.69	21.59	21.50	21.50	6.41
21	13	42	5	"	E. D.	5	21.69	21.70	21.60	21.51	21.51	9.42
21	14		5	"	E. S.	4	21.61	21.61	21.51	21.42	21.42	9.39
23	13	40	5	"	E. S.	4	21.28	21.28	21.18	21.08	21.08	9.31
24	12	15	5	"	E. S.	3	21.17	21.18	21.08	20.98	21.08	9.30
24	12	33	6	"	E. D.	3	21.08	21.09	20.99	20.89	20.89	9.27
29	13	16	5	"	E. S.	3	20.94	20.94	20.84	20.75	20.75	9.46
30	14	50	7	"	E. S.	3	20.78	20.78	20.68	20.59	20.59	9.46
Nov. 2	13	34	5	"	E. S.	6	20.15	20.15	20.05	19.97	19 97	9.36
4	12	40	5	"	E. S.	4	19.85	19.85	19.75	19.67	19.68	9.37
5	13	10	5	"	E. S.	5	19.61	19.61	19.51	19.43	19.44	9.34
5	14	38	5	"	E. S.	3	19.54	19.54	19.44	19.36	19.37	9.30
6	12	45	5	"	E. S.	7	19.67	19.67	19.57	19.49	19.50	9.44
9	13	35	5	"	E. S.	7	19.23	19.23	19.13	19.05	19.07	9.49
14	14	35	5	"	E. S.	6	18.02	18.02	17.92	17.85	17.88	9.34
15	13	53	5	"	E. S.	5	18.03	18.03	17.92	17.85	17.88	9.44
19	12	45	5	"	E. S.	3	17.19	17.19	17.09	17.02	17.06	9.38
20	11	50	5	"	E. S.	2	17.15	17.15	17.05	16.93	17.03	9.46
21	11	45	5	"	E. S.	7	16.86	16.86	16.76	16.70	16.74	9.40

For cor. irr. 0″.10 with power 862 and irradiation varying as the $\sqrt[6]{}$ of the illumination.

Measures made on true diameter.

TABLE II.

EQUATORIAL DIAMETERS.

Time.	h	m	No. of Meas.	Eyepiece		Wt.	Uncor. Meas.	Cor. for Refrac.	Cor. for Irrad on Limb.	Cor. for Irrad on Term.	Cor. for Phase.	Reduced to Distance Unity.
							″	″	″	″	″	″
Oct. 12	13	15	5	862		5	22.15	22.15	22.05	21.99	22.06	9.51
15	14	2	5	"	E. S.	3	21.96	21.96	21.86	21.78	21.80	9.42
17	14	42	5	"	E. S.	4	21.98	21.99	21.89	21.79	21.79	9.44
19	14	48	5	"	E. S.	3	22.00	22.01	21.91	21.81	21.81	9.50
20	14	11	5	"	E. S.	?6	21.80	21.81	21.71	21.61	21.61	9.44
21	14	25	6	"	E. D.	4	21.74	21.75	21.65	21.56	21.56	9.45
21	14	45	5	"	E. S.	5	21.57	21.58	21.48	21.39	21.39	9.37
23	14	15	7	"	E. S.	3	21.45	21.46	21.36	21.28	21.30	9.40
24	11	50	6	"	E. S.	3	21.38	21.38	21.28	21.21	21.24	9.42
24	12	50	5	"	E. D.	5	21.32	21.33	21.23	21.16	21.19	9.40
29	13	3	5	"	E. D.	3	20.91	20.92	20.82	20.77	20.88	9.52
30	15		5	"	E. D.	2	20.54	20.55	20.45	20.40	20.54	9.43
Nov. 2	14	12	5	"	E. D.	6	20.03	20.04	19.94	19.89	20.10	9.42
4	13	10	5	"	E. D.	4	19.87	19.88	19.78	19.74	20.01	9.53
5	13	36	5	"	E. D.	4	19.49	19.50	19.40	19.36	19.66	9.44
5	14	15	5	"	E. D.	3	19.46	19.48	19.38	19.34	19.64	9.43
6	13	20	6	"	E. D.	8	19.59	19.60	19.50	19.46	19.79	9.58
9	13	53	5	"	E. D.	7	19.04	19.06	18.96	18.92	19.34	9.62
14	14	18	7	"	E. D.	6	17.90	17.93	17.83	17.80	18.37	9.59
15	13	36	7	"	E. D.	6	17.69	17.71	17.61	17.58	18.17	9.58
19	13		5	"	E. D.	5	16.76	16.78	16.68	16.66	17.36	9.54
20	12	20	5	"	E. D.	6	16.65	16.67	16.57	16.55	17.28	9.60
21	11	58	4	"	E. D.	1	16.34	16.35	16.25	16.23	16.97	9.53

For cor. irr. 0″.10 with power 862.

Measures made on true diameter.

No. of Meas. denotes the number of the differences in the readings between the movable and the fixed wire, the movable one having been placed first on one side of the fixed one, and then on the other.

Before discussing the table the corrections in the several columns deserve notice. Of these, the first is that arising from refraction. This is the correction due to the differential effect of refraction upon the planet's opposite limbs at the extremities of the particular diameter measured. It depends upon the altitude of the planet at the time of observation, and upon the inclination at that moment of the particular diameter to the vertical. In about half the measures only was it large enough to make itself perceptible to hundredths of a second of arc.

The correction for abberation, similarly a differential effect, was so utterly insignificant throughout as not to appear at all.

The next correction is that due to irradiation. As this is the chief correction into which uncertainty enters we will consider it at some length, the more so that no investigation of it, has, to my knowledge, ever been made. It is an important correction but one for which no possible value can destroy the quantities upon which the evidence for the twilight rests, or even effect appreciably the amount of the polar flattening. This will be evident farther on. Its effect is primarily upon the absolute values of the diameters; and secondarily upon the amount, not the existence, of the twilight arc.

It is to be noticed, to start with, that the irradiation is a function of the illumination and of the observer. For a given observer, therefore, it is a function of the illumination, and for a given illumination it is a personal constant. It is independent of the size of the object and diminishes relatively with the magnification. Other things equal, the greater the power used, the smaller the correction arising from it.

Toward its determination two different tests were made, in each case upon both Professor W. H. Pickering and myself; in the one the effect should have been less than in the case of Mars, in the other greater. As in both cases the observers substantially agreed, the results may be accepted as having some impersonal value.

The first test was made upon a railroad switch-head, a white circular disk with a smaller black circle painted upon it. The size of these circles was unknown to the observers.

Their estimates were

(W. H. P.)	(white rim)	1.; (diameter black circle)	1.3
(P. L.) (two sets—mean)	"	1.; " " "	1.265

The disks and their distance were then measured and gave:

for diameter black circle : 202 mm.
for radius white rim : 126 mm.
for ratio : $\frac{1}{16}$
for distance from eye : 57 yds.
Therefore 1 mm. equalled 3″.9

For the amount of the irradiation in seconds of arc, x, assume the amount of the irradiation of the white rim against the general background of earth of a brown color to have been ⅔ of that of the rim against the black circle. We have, then, for the first observer, the following equation to determine x.

$$\frac{252 \text{ mm.} + \frac{1}{3}^{0}\, x}{212 \text{ mm.} - \frac{2}{3}\, x} = \frac{2.0}{1.3}; \text{ from which } x = 9.2 \text{ mm. or } 36''$$

for the second observer $x =$ 40″

The second test was on the Moon (Nov. 22, 1894), when the old moon was seen on the new moon's arms. In this case the irradiation proved for both observers to be 1/7 of the radius of the old moon or about 157″.

The illumination of the surface of Mars bears to the illumination of the lunar surface the ratio $\frac{1}{(1.5)^2} \cdot \frac{26}{17}$ to unity; the first factor denoting the ratio due to their respective distances from the Sun, and the second that due their respective albedoes. In round numbers this = $(2/3)^2$ 3/2 or 2/3. But in the telescope, owing to the loss of light in passing through the lenses, first of the object glass and then of the eyepiece, this is further to be diminished by the ratio of .82 to unity; which gives .55 for the relative illumination of the Martian surface as compared with the lunar.

Irradiation being a question of contrast and the contrast between the Martian limb and the sky being pretty certainly greater than that of the white rim and the black circle of the switch-head, just as it is certainly less than that of the Moon's bright limb and the sky, to which the contrast between the limbs of the old and of the new Moon closely approximates, the value for Mars should lie between the two determinations. Just where it falls between the two is a matter of estimate inasmuch as the exact functional relation between the illumination and the irradiation is not known. If we assume it to be, for short distances from the edge of the bright body, as the square root of the illumination, we have the irradiation from the Martian disk in the telescope 3/4 of that of the lunar to the naked eye; if as the fourth root .87, if as the sixth .91. Perhaps we shall not be far out if we take .8 for the value. This gives 127″ for Mars. But this again should be slightly diminished on account of the lightening of the field in the telescope due to the inevitable scattering of light by the surfaces of the lenses and the diminished contrast in consequence. If we assume the field to be relatively twice as bright for Mars, we get about 110″ for the irradiation. With a power of 862,110″ becomes 0″.125. For a lower power it is proportionately greater, for a higher one proportionately less. All the measures on and after September 23d were made with a power of 862; those before with powers from 440 to 1308. I have calculated two tables, one on the basis of an irradiation of 0″.10 and one on that of 0″.15; the first table is given in its entirety, of the second only the means. Lastly, account has been taken of the time at which the measure was made, whether dur-

ing the night, the dawn, or the daylight, and the correction applied in accordance. Such is the correction for irradiation upon the planet's limb. The double of it, therefore, would need to be subtracted from the measures of a disk similarly placed to that of Mars and fully illuminated.

But here we come to a correction upon this correction. Except at the moment of opposition the disk of the planet showed, not two bright limbs, but a limb and a terminator. Now it is evident at once that the irradiation upon the terminator must be very different from that upon the limb, inasmuch as the light fades away to nothing at the one while it has its full value at the other.

To determine the amount of the consequent correction needed at the terminator, it is to be observed that if Mars presented a dead surface like a plaster cast, we should have the edges less brilliant than the centre and the following formula to express the difference

$$m\sqrt{c}\ (\sin \alpha)^{\frac{1}{n}} - [\cos \gamma - \cos (\alpha + \gamma)]$$

If again the illumination were equal all over we should have for values of the phase angle appearing in these measures:

$$m\left(\frac{\sin \alpha}{\sin (\alpha + \gamma)}\right)^{\frac{1}{n}} - \Big[\cos \gamma - \cos (\alpha + \gamma)\Big]$$

where γ is the areocentric angle between the Sun and the Earth; α the angle between the terminator and the point of the illuminated surface whose irradiation is sought; m, the ratio of the irradiation at the limb to the radius of the disk; n, denotes the ratio of the irradiation to the illumination; and c is a constant determined so as to make the irradiation at the limb unity.

But the actual surface of Mars is neither of these, being actually brighter near the limb than elsewhere. If this effect were due, as it probably is, to illuminated particles in the air, the first term would depend, omitting the effects of temperature, etc., upon

$$n\sqrt{\int_{\varphi_1}^{\varphi_2} \sec^2 \varphi,\ c,\ e^{-\alpha \sec \varphi}\ d\varphi}$$

where the limits φ_1 and φ_2 depend upon the chord intercepted between the top of the atmosphere and the tangent to the surface at the point, and the integral is the amount of atmosphere necessary for visibility. The term is then to be multiplied by the ratio of the illumination of the disk at the time to that at opposition.

The integral cannot be directly found, being attainable only by a series. As we do not know that this fully expresses the fact, it will be sufficient to estimate approximately the amount of the correction for each varying phase angle. This will be found in column 9. Substitution in the column of any other value, even to the whole extent of the correction for irradiation at the limb—a value which would imply equal illumination there, which is not only theoretically impossible, but visibly not the case—would not suffice to affect, substantially, the result, as will appear on performing the operation and comparing the values obtained with those given below.

The corrections for irradiation at the centre of the limb and at the extremities of the phase diameter have been taken as the same. This would be approximately true in the case of an atmospherically enveloped planet. It would not be true for a bare one, as is evident from the case of the Moon. When the Moon shows a crescent the greater irradiation at the center of the limb is manifest to the naked eye in its distortion of the limb beyond a perfect semi circle. The above consideration has of course no effect upon the question of a twilight arc; but only upon the polar flattening.

Fourth comes the correction for phase. Inasmuch as the phase axis and the polar axis did not in general coincide, there entered into its determination beside the amount of the lacking lune, the angle of inclination of the two axes. So that the amount of the defalcation had to be calculated in accordance for each night.

Fifth comes the correction for tilt. As the planet's polar axis was inclined to the Earth, the diameter measured for the polar one was not the true polar diameter. The diameter measured for the equatorial diameter, although also not in fact an equatorial diameter, was always exactly equivalent to one since its extremities were always 90° distant from the pole. To get the correction to be applied to the polar diameter, the polar flattening was taken at 1/200, a value sufficiently near the truth, and gave 0″.007 at the beginning of the measures, 0″.006 in the middle, and 0″.007 again at the end. This being, for the interval of a few days practically constant was applied to the several means into which the resulting columns were divided. It does not therefore appear in Table I.

No correction is needed for astigmatism, since the measures themselves correct it, inasmuch as image, field, and wire are all three proportionately affected.

In the last column appear the measures reduced to distance unity.

So soon as the measures had been corrected and reduced, two results made themselves evident, both so large as to be almost unmistakeable, before taking the means. One was the polar flattening. This was more or less expected. The other was not; but it was even more noticeable. It was a systematic increase in the size of the equatorial diameter from opposition to the times when the last measures were made in November. The November measures were evidently larger than the October ones; while, on the other hand, the polar measures showed no corresponding increase. Struck by this fact and suspecting its cause, instead of taking the mean of all the measures for each diameter, I divided them into sets according to their proximity in date to the time of opposition, and took the mean of these sets.

Allowing for tilt as well as for the other corrections the means are as follows:

MEASURES POLAR DIAMETER.

Mean Oct. 15th to Oct. 23d both dates inc.	9″.379
" " 12th, 24th, 30th, both dates inc.	9 .375
" Nov. 2d to Nov. 21st, both dates inc.	9 .390

MEASURES EQUATORIAL DIAMETER.

Mean Oct. 15th to Oct. 23d both dates inc.	9″.420
" " 12th, 24th and 30th both dates inc.	9 .457
" Nov. 2d to Nov. 21st both dates inc.	9 .545

Opposition occurred on October 20th. The first set in each schedule, therefore, was made within four days of opposition; the second, within eleven days of it; the last from fourteen to thirty-two days after it. That there is a systematic increase in the equatorial measures is apparent. That it is not paralleled by a corresponding increase in the polar ones shows instantly that it can hardly have been due to systematic error in the observer, since in that case both sets of measures would in all probability have been affected.

Now as all the measures had previously been corrected for refraction, irradiation, phase and tilt, the means of each diameter should have agreed with themselves. The polar did so in a very satisfactory manner; the equatorial not only did not, but they differed in proportion to the distance in time from the date of opposition. Now the only factor that increased in proportion to the distance in time from opposition was the phase. The direct effect in the way of decreasing the equatorial diameter had already, as we have seen, been allowed for; what is more it is a correction susceptible of some exactness, since it depends upon the motions and relative distances of the Earth and Mars, quantities very accurately known. Besides these quantities there is

nothing which enters into the calculation but the position of the pole of Mars, and this would have to be, not only some 35 Martian degrees in error to explain the discrepancy, but, as we shall see later, would have had to have shifted obligingly to an opposite error during July and August to account for the measures taken then. In other words no such discrepancy exists.

In the case of a bare globe this direct effect would be the only effect phase could have upon the equatorical diameter; not so, however, in the case of a body not bare. If a planet possessed an atmosphere, that atmosphere would cause the phenomenon of twilight, and to an observer at a distance the effect of the twilight would be to prolong the terminator beyond what would otherwise be its limit. There would thus result a seeming increase in the equatorial diameter as the disk passed from the full to the gibbous phase. Not only would this be the case, but contrarywise in the absence of an atmosphere, the measures of the equatorial diameter, should not only have shown no increase with increase of phase, but should actually have shown a decrease, inasmuch as it would be impossible for an observer to see quite out to the edge of the disk under its diminishing illumination.

TABLE II.—EARLY DIAMETERS.

POLAR.

Measured by Mr. A. E. Douglass.

	Time.		No of Meas.	Eye-piece.	Wt.	Seeing Set on Phase Axis	Uncor. Mean.	Cor. Irr. on Limb.	Cor. Irr. on Term.	Cor. for Refrac.	Cor. for Phase.	At Dist. Unity Appar.	At Dist. Unity True.
	h	m					"	"	"	"	"	"	"
July	6	17 10	4	617	8	5	12.12	12.07	12.02	12.03	12.03	10.01	10.00
"	8	16 42	4	"	10	3	12.83	12.76	12.69	12.69	12.69	10.01	10.40
"	20	16 10	3	"	6	2	13.02	12.95	12.88	12.89	12.89	9.68	9.67
"	22	18 8	5	1305	6	3	12.93	12.92	12.90	12.90	12.90	9.54	9.53
Aug.	11	16 10	5	"	6	4	14.97	14.94	14.90	14.91	14.91	9.47	9.46
"	14	14 56	5	"	7	4	15.31	15.24	15.17	15.18	15.18	9.40	9.39
"	21	17 55	5	617	7	6	15.92	15.88	15.84	15.84	15.84	9.25	9.24
Sept.	20	14	5	"	4	3	20.59	20.45	20.31	20.32	20.32	9.43	9.42
"	23	14 35	5	862	5	3	20.85	20.75	20.65	20.66	20.66	9.42	9.41
Oct.	5	14 36	7	"	4	3	21.77	21.67	21.57	21.58	21.58	9.37	9.36
							EQUATORIAL.						
July	6	17	4	617	6	5	9.93	9.88	9.88	9.88	11.74	9.76	9.76
"	8	16 25	6	"	6	3	10.49	10.42	10.42	10.42	12.39	10.15	10.15
"	18	18 30	5	"	6	5	10.67	10.64	10.64	10.65	12.56	9.58	9.58
"	22	17 55	5	1305	8	3	10.75	10.73	10.73	10.74	12.66	9.38	9.38
Aug.	11	15 45	5	"	6	3	12.79	12.74	12.74	13.74	15.83	10.04	10.04
"	14	14 45	5	"	7	5	13.79	13.72	13.72	13.72	15.74	9.75	9.75
"	21	17 45	5	617	7	6	14.02	13.98	13.98	13.99	15.85	9.26	9.26
Sept.	20	14 15	5	"	5	3	19.69	19.55					
"	23	14 12	5	362	6	3	20.11	10.01					
Oct.	5	14 15	7	"	4	4	21.83	21.73					

For irr. 0".10.

The effect which would be due to a visible twilight is thus strikingly reflected in the measures. Nor is this all. Comparison with the measures made in July, August, and September and the first part of October brings the same effect even more prominently into view. The table of these measures is opposite.

Now if we take the means of these measures also in chronological sets and put them beside the later ones, we get the following most curious corroboration:

TABLE III.—MEANS.

	Cor. for Ref. Irr., tilt and phase. Irr. 0″.10	Prob. Error. ″	Angle of Phase. °	Irr. 0″.15
POLAR DIAMETERS.				
July (6 to 22 inc.)	9.976	0.13	0	9.933
Aug. (11 to 21 inc.)	9.362	0.04	0	9.325
Sept. (20 to Oct. 5 inc.)	9.401	0.012	0	9.355
Oct. (12 and 24 to 30 inc.)	9.375	0.028	1	9.336
Oct. (15 to 23 inc.)	9.379	0.011	2.5	9.339
Oct. (12 and 24 to 30 inc.)	9.375	0.028	1	9.336
Nov. (2 to 21 inc.)	9.390	0.012	4	9.350
EQUATORIAL DIAMETERS.				
July (6 to 22 inc.)	9.691	0.11	46.5	9.672
Aug. (11 to 21 inc.)	9.666	0.15	41	9.645
Sept. (20 to Oct. 5)	9.523	0.010	20.5	9.490
Oct. (12 and 24 to 30 inc.)	9.457	0.016	7	9.417
Oct. (15 to 23 inc.)	9.429	0.010	1	9.385
Oct. (12 and 24 to 30 inc.)	9.457	0.016	7	9.417
Nov. (2 to 21 inc.)	9.545	0.015	19	9.514

The effect here is certainly striking. On the one hand we see that except for July the values of the polar diameter remained substantially the same from beginning to end; on the other hand, that the equatorial diameter increased on *both* sides of opposition, that it increased in proportion to its distance from opposition, and that the probable errors of the means are much less the values of the variations.

The smallness of the polar means for October 12th, 24th and 30th is doubtless to be attributed to the fact that some of the measures then taken were abnormally small.

That the probable errors are so much larger in the earlier measures is partly explained by the relatively small size of the disk at the time; for the probable errors would vary inversely as the square of the disk's diameter.

By way of applying still further correction to the above result we may now take into account the position of the observer's eyes although the consequent correction is too small to be practically appreciable.

The result is not without interest on its own account, for it dis-

closes a sort of psychic analogae to astigmatism. Astigmatism proper cannot enter into the values obtained because it must affect equally the space passed over by a turn of the micrometer screw and the image itself. It therefore cancels out of the result. Nevertheless, measures of double stars show a difference according as the distance between them is measured vertically or horizontally to the line joining the observer's eyes. To determine whether there is any such effect in the present case we notice that among the measures are four pairs made under identical conditions except for the manner of setting the eyes. These pairs are as follows:

Polar Diameters.				Equatorial Diameters.			
	h m				h m		
Oct. 21	13 42	E. D.	9.42	Oct. 21	14 25	E. D.	9.45
Oct. 21	14 00	E. S.	9.39	Oct. 21	14 45	E. S.	9.37
Oct. 24	12 15	E. S.	9.30	Oct. 24	11 50	E. S.	9.42
Oct. 24	12 33	E. D.	9.27	Oct. 24	12 50	E. D.	9.40

On scanning these there seems to be no preponderance one way or the other since in just half the pairs the E. D. measures are the greater and in half of them the reverse. But on further inspection it appears that another personal factor is at work; for on considering the matter of time it turns out that in every pair the later measure is the smaller. If from the hint thus given the

	Equatorial Diameter. Taken before or after Polar Diameter.	Equa. diam. greater than polar one by "	"	"
Sept. 20	after	+ 0.10		
" 23	before		+ 0.08	
Oct. 5	before		+ 0.18	
" 12	before		+ 0.11	
" 15	after	+ 0.06		
" 17	before		+ 0.07	
" 19	before		+ 0.04	Difference beween the means of those taken before and after + 0.03 those taken before being this much larger than those taken after.
" 20	after	+ 0.03		
" 21	after	+ 0.06		
" 21	after	+ 0.01		
" 23	after	+ 0.09		
" 24	before		+ 0.12	
" 24	after	− 0.13		
" 29	before		+ 0.06	
" 30	after	+ 0.03		
Nov. 2	after	+ 0.06		
" 4	after	+ 0.16		
" 5	after	+ 0.10		
" 5	before		+ 0.13	
" 6	after	+ 0.14		
" 9	after	+ 0.13		
" 14	before		+ 0.25	
" 15	before		+ 0.14	
" 19	after	+ 0.16		
" 20	after	+ 0.14		
" 21	after	+ 0.13		
		mean + 0.09	mean + 0.12	

whole polar series be compared, as regards priority of observation, with the whole equatorial one, measure for measure, the same difference is brought to light, as appears on taking the mean of the foregoing table.

Turning now to the measures of the artificial disk upon Mt. Agassiz as observed by Mr. Douglass, we have:

DISK.

VERTICAL DIAMETERS.

	h	m	Direct Meas.	Seeing	Wt.			Cor. for Refr.
								″
Oct. 25	5	19	"	"	3	5	E. D. 20.87	20.92
Oct. 25	5	33	"	"	2	3	E. D. 21.11	21.16
Oct. 31	4	36	"	"	2	3	E. D. 21.16	21.21
Nov. 6	5	21	"	"	2	3	E. D. 20.83	20.88
Nov. 8	5	3	"	"	½	6	E. D. 20.97	21.02
Mean from Seeing							20.98	21.03
Mean from squares of Seeing							20.96	21.01

HORIZONTAL DIAMETERS.

	h	m					
							″
Oct. 25	5	5	Direct Meas.		5	6	E. S. 21.09
Oct. 25	5	45	"	"	2	2	E. S. 21.14
Oct. 31	4	19	"	"	2	3	E. S. 21.36
Nov. 6	5	9	"	"	2	2	E. S. 20.91
Nov. 8	4	42	"	"	1	7	E. S. 21.01
Mean from Seeing							21.11
Mean from square of Seeing							21.10

from which we find for the horizontal diameters taken before or after the vertical diameter.

Oct. 25	before	difference + .17		difference.
Oct. 25	after		− .02	
Oct. 31	before	+ .15		0″.105
Nov. 6	before	+ .03		in favor of before.
Nov. 8	before	− .01		

but as this was on a disk of 21″ it becomes on one of 9″.4 about 0″.05. This agrees within expectation with the result found from Mars.

Returning to the four pairs of measures of Mars and using the value 0″.03 deducted from the whole series of measures of the planet for the before-after correction we get 0″.02 for the correction between E. S. and E. D., E. D. being = E. S. + 0″.02.

For corroboration of this value it is not possible to use the measures of the artificial disk. For inspection of them shows that the observed measures were actually smaller than the real ones, which hints that the disk, in consequence of exposure to the weather, shrank. It becomes, therefore, probable that the disk shrank unequally and therefore its actual dimensions become unreliable.

If now we apply both these corrections to the previous table of

the means of the two diameters we have, taking for basis E. S.:

	Polar.		Equatorial.	
	Irr. 0″.10		Irr. 0″.10	
July	9.976	9.98	9.691	9 69
August	9.362	9.36	9 666	9.67
Sept. 20, 23, Oct. 5	9.406	9 41	9.518	9 52
Oct. 12. 24, 30	9 379	9.38	9.452	9.45
Oct 15, 23	9.379	9 38	9.429	9.43
Oct. 12, 24, 30	9 379	9 38	9.452	9 45
Nov. 2, 21	9 380	9.38	9.540	9.54

The same relative result would have come out in the table for the irradiation correction 0″.15.

Professor W. H. Pickering's measures, although relatively few in number, confirm so far as they go Mr. Douglass'. They are as follows:

TABLE IV.—EARLY DIAMETERS.

Measured by Professor Pickering.

	Time.		Means.	Eye-piece.	Wt.	Setting set on Phase axis	Uncor. Means.	Cor. Irr. on Limb	Cor Irr. on Term	Cor. for Refrac.	Cor.for Phase.	At Dist. Appar.	Unity True.
	h	m											
							Polar.						
							″	″	″	″	″	″	″
July 11	16	52	5	617	4	3	12.36	12.33	12.30	12.31	12.31	9.88	9.87
21	18	40	5	"	5	4	12.98	12.96	12.94	12.94	12.94	9.64	9.63
Aug. 7	17	52	5	"	7	7	14.16	14.14	14.12	14.12	14.12	9.30	9.29
24	15	20	5	"	5	4	16.67	16.60	16.53	16.54	16.54	9.44	9.43
							Equatorial.						
July 11	17	10	5	617	3	3	10.54	10.50	10.50	10.50	12.47	10.01	10.01
21	18	30	5	"	5	4	10.91	10.88	10.88	10.89	12.84	9.58	9.58
21	18	48	5	"	5	4	11.08	11.05	11.05	11.06	13.04	9.73	9.73
Aug. 7	16	28	3	"	7	7	12.62	12.55	12.55	12.55	14.55	9.52	9.52
24	15	5	5	"	4	4	15.06	14.92	14.92	14.92	16.80	9.59	9.59
28	18	30	5	"	3	3	15.25	15.21	15.21	15.22	16.97	9.36	9.36

TABLE V.

MEANS.

Professor W. H. Pickering.

Polar Diameter reduced to dist. unity (all corrections made except for the twilight band).....9″.373

Equatorial Diameter reduced to dist. unity (all corrections made except for the twilight band).....9″.580

Polar Diameter, polar cap away from limb.....9″.14

Polar Diameter, polar cap on the limb.....9″.49

Twilight arc 11°.

To determine the extent of the twilight thus disclosed by the measures: if we call E the phase angle; T the angle between the radius to the sunset point and the radius prolonged, to the point of the atmosphere last illuminated; *a*, the true equatorial radius reduced to distance unity, and *b*, the corresponding amount of its excess, we have the following equation to determine T:

$$\frac{1 + \cos E + \sin E \,.\, \tan T}{1 + \cos E} \,.\, 2a = 2a + b$$

T, neglecting temperature, etc., being determined from

$$\int_{\varphi_1}^{\varphi_2} \sec^2 \varphi \; C \,.\, E^{-a \sec \varphi} d\varphi$$

where the limits φ_1 and φ_2 are such as to make the integral constantly equal to the amount of illumination necessary for visibility. Whence

$$\tan T = \frac{b \,.\, 1 + \cos E}{2a \,.\, \sin E}$$

As b is the mean value of the excess of the equatorial diameter between certain times, we must, to determine T take the mean value of

$$\frac{1 + \cos E}{\sin E}$$

between the same times.

This mean value is in form,

$$\int_{E_1}^{E_2} \frac{1 + \cos E}{\sin E} dE$$

But as dE is not constant if dt be taken constant, its value in terms of dt must be introduced from the elliptic motions. A short cut to the result may be taken, however, by deducing this mean value direct from the phase corrections.

By so doing we get an approximate value of T, which applied to the measures of October 15th to 23d—since even these were not made exactly at opposition—gives a new value for b and a closer approximation to $2a$. By repeating this process we get values for both as near as we please to their true values until $E = T$. After this, when $E < T$, the equation ceases to be true, b diminishing not to nothing, but to a minimum with irradiation 0″.10 of about 0″.025 at distance unity, the amount namely of the twilight seen in profile on the limb. This also is affected by irradiation, since the twilight itself would be swamped in the irradiation effect of the limb, and at the same time would cause an irradiation of its own. We can thus get an approximate value for the twilight arc which is evidently about 5°, the double of which or 10° is the angle which determines the duration of the twilight. On the Earth this angle is 18°.

The value for the twilight band deduced from these observations does not measure the full breadth of that band. It gives

only a minimal value for it. For although Mars shows us a disk which is always more than half-full, in which aspect an illuminated fringe of atmosphere would be more perceptible to an observer placed without than to one placed within it, provided both were at the same distance off, in the case before as the observer is a great deal farther off. In consequence, what would be quite recognizable to one standing upon the planet's surface would be too faint to be seen by him at a distance of forty millions of miles. The detection, therefore, of any twilight on Mars hints that the extent of that twilight is greater than appears; how much greater we cannot at present say.

A second possible cause affecting the extent of the twilight is the constitution of the Martian atmosphere. That atmosphere is practically cloudless; if also it be clearer than our own, the twilight would be relatively less, for the amount of twilight is, among other things, a question of the clearness of the air. It is the particles suspended in the air that reflect the light; the less particles, therefore, the less the twilight.

Polar Flattening.

From these measures we deduce for the polar flattening $\frac{a-b}{a}$ the value 1/190

This value receives corroboration from two other sources. It is, in the first place, happily accordant with what theory would lead us to expect. Tisserand has found that with the known rotation of Mars and supposing homogeneity, the planet's flattening should be 1/175 of the equatorial diameter, while if the strata varied in density after the manner of those of the Earth, the polar flattening should be 1/227 of it. Now assuming Mars to have been developed in general accordance with nebular hypothesis, his strata would neither be homogeneous on the one hand, nor on the other, would they vary in density from the surface to the center so marked as is the case with those of the Earth. For Mars, being a smaller body, the pressure due to gravity would be less, somewhere between that of the Earth and that of homogeneity (which is nothing) and the polar flattening should therefore be somewhere between 1/227 and 1/175 of the equatorial diameter. 1/190 is, therefore, not far from the value probable *a priori*.

Secondly, since this paper was written Hermann Struve has found from the speed of rotation of the apsides of Deimos and Phobos the value 1/190 for the polar flattening. It is interesting to have this result agree thus closely with theory, as it furnishes

so much more reason for believing in the general evolution of our solar system.

Any value much less than 1/190 would require that Mars should have solidified an unconscionably long time ago. For as we see, the Earth still rotates very nearly as she did when she solidified, and although in the case of Mars this must have happened relatively long before, yet on the other hand, Mars lacks relatively the tools for the purpose, possessing neither suitable satellites on the one side, nor sufficient seas on the other.

For the absolute dimensions of the diameters we may best take the correction for irradiation at 0″.125 and thus obtain:

Polar Diameter..... .....9″.32
Equatorial Diameter.....9 .37
Ellipticity.....$\frac{1}{190}$
Twilight arc (minimal value) about 10°

LOWELL OBSERVATORY,
Dec., 1895.

ALMANACS. V.

R. W. McFARLAND.

FOR POPULAR ASTRONOMY.

At the close of the last article it was intimated that a few words on the chronological part of an almanac, would be sufficient. On further reflection, and after correspondence with the editor, it is deemed best to enter a little more fully into the subject, as is here done:—and it is further hoped that the facts here set forth may be acceptable to many readers of this journal.

Under the heading "Chronological Cycles," there are usually set down six particulars, viz. Dominical Letter, Epact, Solar Cycle, Golden Number (sometimes called Lunar Cycle), Roman Indiction, and Julian Period. These will be explained in full. After these items, most almanacs give the date for Easter and of other feasts or festivals to the number of eight, ten, or twelve,—some being fuller than others in regard to this matter. As Easter moves forward or backward, they all move with it: wherefore it was a great blunder for an author to say, in an otherwise excellent work on astronomy, that "This is the solitary religious festival, which, in Christian countries, depends on the motion of the Moon." Below I give *thirteen* others, without exhausting the number.

DOMINICAL LETTER.—Call January 1st, A; 2nd, B; 3rd, C; etc.; G will be the seventh day; begin again with A for the 8th, B for

the 9th, and so on for the whole year: the letter which stands for Sunday is the *dominical* or Lord's day letter. It is used in determining the date of Easter. The year 1895 began on Tuesday, which is represented by A, according to the preceding explanation. Counting forward, Sunday is represented by F, the *dominical* letter for that year. This year, 1896, began on Wednesday, and by a like course, one can easily see that the letter for Sunday is E. This is for the first two months,—D being the letter for the last ten months. Here is the explanation of the change.

We use, substantially, the calendar made by Julius Cæsar, and which went into effect January 1st, 45 years "before Christ," i. e. 45 B. C. The Roman method of designating the days of the months was wholly different from ours, and was far less simple. All classical scholars are supposed to understand it. The first day of the month was called the "Calends," i. e., March 1st was the Calends of March. The 28th of February was called "the day before the Calends of March;" the 27th of February was called "the third day before the Calends,"—the first of March being always reckoned as one of the three. In this way the 24th of February was the "sixth day before the Calends." There was, by this ecclesiastical method of counting, no 29th of February, as we now have it: but this extra day of leap year was allowed for by counting the 24th *twice*. The Latin words for *twice*, "bis" and *sixth*, "sextilis" give us the word "bissextile," a word denoting not only this second 24th, but also the year in which this second 24th occurs. In 1896 the 23rd of February falls on Sunday, which, as shown above, is represented by E. Monday, the 24th, is represented by F; and by the ecclesiastical way of proceeding, Tuesday is not the 25th, but is the *second* 24th, and the letter F stands for this second 24th also. Then Wednesday is the 25th, and is denoted by the last letter G; Thursday the 26th is A; Friday, 27th is B; Saturday, 28th is C; and Sunday, March 1st is D. All the remaining Sundays of the year will be denoted by D. March 1st must always be represented by D, when January 1st is A.

The year consisting of fifty-two weeks and one day, a common year begins and ends on the same day of the week:—wherefore the Dominical letter will be one higher in the alphabet; i. e. if it is D one year, it will be C the next; the order of recurrence being always from G back to A.

The Epact. This is the Moon's age January 1st, counting according to the method established by the Council of Nicæa, in 325 A. D. If the new moon should occur on December 30th, by

January 1st, the Moon would be two days old, and the Epact would be 2. But the new Moon determined by the arbitrary rule laid down by the aforesaid council may differ two or three days from the actual new Moon; but this difference is of no moment, the original rule having been made in order to have one definite and easily determinable day for the celebration of Easter.

Easter is the Sunday following the full Moon which, by the Council's order or rule, happens on, or next after March 21st. Should the full Moon occur on Saturday, March 21st, the next day would be Easter. The 22d of March then is the earliest possible date for Easter. Nor can it occur later than April 25th. The Moon's age March 1st is always the same as it is January first, and knowing the Epact, we are on the direct road to find the 14th of the lunar month—this 14th being reckoned as the date of the full Moon. The process will be given further on.

SOLAR CYCLE.—This is a period of 28 years, after which the days of the week on which any given day of the month happens, will return again in the same order. Did the year consist of exactly 365 days, the order of change would finish its course in 7 years; but each fourth year, in general, being a leap year, there is a jump of one day every fourth year. But taking the product of 7 and 4,—28, we have the Solar Cycle, as above defined. Wherefore, any day of any month must fall at least once on every day of the week within 28 years. It may occur several times.

Before the middle of this century, and even later, it was frequently said, in the newspapers, that the reason why the 4th of March was selected for the President's inauguration, was, that the 4th would not occur on Sunday for 300 years This was considered an infallible proof of reverence for the Sabbath, on the part of the Congressmen who fixed that date. But if their piety had no more solid foundation than this, then it would share the fate of

> * * "The baseless fabric of this vision,
> The cloud-capp'd towers, the gorgeous palaces,
> The solemn temples, the great globe itself,"

which, it is said, should disolve, and

> "Leave not a rack behind."

It is allowable to think that no such reason ever troubled the consciences of Congressmen, then or now.

GOLDEN NUMBER.—This is a cycle of 19 years, after which the new Moons occur again on the same day of the month, and almost at the same hour of the day. The discovery of this fact

is attributed to Meton, an Athenian astronomer, more than 400 years B. C. It is sometimes called the Lunar Cycle.

ROMAN INDICTION.—In the early days of mankind, it is not probable that there was any concern at all about dates, or seasons, or years. Herodotus is called the father of history, and his history does not contain a single date. Substantially the same thing may be said of Thucydides, who wrote only a little later—somewhat over 400 years B. C. If Geography and Chronology are the two eyes of history, then some histories are blind of one eye, and see but little out of the other. It seems not to have occurred to any one, in those long-gone-by ages, to invent or use a standard of continually increasing years. The earliest cycle was the Olympiad, a term of four years, used in Greece three centuries and more before the time of Herodotus. The Indiction was a period of 15 years, invented 1000 years after the beginning of the Olympiads. It was first used by the Greek Emperors, and the invention is ascribed to the Emperor Constantine, and the date given as January 1st 313, A. D.

JULIAN PERIOD.—Like the preceding, this is not an astronomical period at all. It was proposed by Joseph Scaliger, and printed in 1583, in a work entitled *De Emendatione Temporum*. It has been long accepted by astronomers and chronologists, as a fixed standard to which all dates may be referred. It is the continued product of 28, the Solar Cycle; 19, the Golden Number; and 15, the Roman Indiction, and is 7980. This, too, is a cycle but one so long that it answers as well as one that should continue to infinity.

The Indiction runs out in 15 years, then begins at 1 again, and repeats itself. Similarly, the Golden Number ends with 19, and begins again; and the Solar Cycle, at 28. These three cycles are supposed to begin at the same time; and the year 1 of the Julian Period is the year 1 of all the others. When was that time? I will show.

The Indictions commencing in 313, the year 312 must have been No. 15 of the preceding cycle, if there had been a cycle. Dividing by 15, we have a certain quotient and a remainder of 12. This shows that the most distant cycle lacked 3 of being full, at the beginning of the Christian Era; and this is equivalent to saying that 3 years of the cycle had passed at the beginning of the year 1. In like manner, we may take the Golden Number for any year, and determine in what year of the cycle, the year 1 of our era fell. The Golden Number for 1895 was 15. Counting backwards, it is readily seen that the Golden Number was 1 in the

year 1881. It follows that it was 19 in 1880. Dividing 1880 by 19 we get a certain quotient and a remainder of 18, lacking 1 of a complete cycle. Wherefore the year before the beginning of our era, had 1 for its Golden Number. For 1895 the Solar Cycle was 28. Dividing 1895 by 28, we find a certain quotient, and a remainder of 9; so the year before the Christian era the Solar Cycle was 9. Hence the rules which we find for determining these cycles. For the solar cycle, add 9 to the year; and divide by 28; the remainder is the cycle for that year. For the Golder Number, add 1 and divide by 19; the remainder is the Golden Number. In like manner, for the Indiction, add 3 and divide by 15, and use the remainder. Any one can test the correctness of these rules by taking any year and trying. This year 1896 would give 1, 16, and 9 for the above cycles. These numbers have never before occurred together in any year since the earliest date of recorded history; and it will be 7980 years before they all appear together again.

It is a very pretty problem in Indeterminate Analysis to determine the year of the Julian Period when the other three cycles are given. Taking them for 1896 as above, and making the Calculation, we find the Julian Period for 1896 to be 6609—as is printed in the Almanacs. One accustomed to such work, can make the computation in a few minutes. [Indeed, as an experiment, after writing the preceding sentence, I took the cycles for the first year of the Christian Era, Solar 10, Golden 2, Indiction 4, and made the computation in *two minutes and a half*, the resulting number being 4714.]

In the 4th edition of Herschel's Astronomy a rule is given for finding the year of the Julian period, but is given without proof. The same rule was given in Hutton's translation of Delambre's edition of the Mathematical Recreations of Ozanam, printed in 1814—also without demonstration; but with a demonstration of a different rule. The old rule, like the men at Bunker Hill on a certain occasion, came "down to us from a former generation."

The Epact depends on the Golden Number. For any year of the present century, the Epact is found by deducting 1 from the Golden Number, multiplying the remainder by 11, and dividing by the 30, the days in a month. Twelve lunar months make 354 days, 11 less than a year. If then we have the Epact for one year, add 11, we have the Epact for the next. For 1896 the Golden Number is 16; deduct 1, multiply the remainder by 11, and throw out the months, the remainder is 15, which is the Epact for 1896. For the next century the same rule will apply, except that the final remainder as given above must be dimin-

ished by 1. The reason is that the year 1900 loses one day because it is not a leap year. But a full and complete explanation would lead us too far astray.

Easter depends on the Epact and the Dominical Letter.

For finding the Dominical Letter there are several rules; some simple, some complex. The simplest which I have used is this: to the year add its fourth part, rejecting all fractions; divide the sum by 7, to throw out the weeks; if the remainder is 1, the letter is G; if 2, it is F; if 3, it is E; and so on up to 7 or 0 which is A. It will be remembered that the Dominical Letters run upward in the alphabet. The one-fourth of the year is added, because one-fourth of the years are leap years. The same rule will hold for the next century, if we subtract one from this sum before dividing by 7. This 1 comes in because 1900 is not a leap year.

There are also several ways of finding Easter. The easiest way is to pick it out from the tables where it is, in effect, given for several thousand years. But then one must have the tables and know how to use them.

The most convenient method (for those not possessing the tables) is developed below. The lunar month in which Easter occurs is called the Paschal moon. Its 14th day is taken as the time of the full Moon. Easter is the first Sunday following the full Moon which occurs on, or next after March 21st. If March 21st is Saturday and the full Moon occurs then, the next day is Easter. So March 22d is the earliest possible date for Easter. The latest possible day is April 25th. When the full Moon is on the 21st then Easter is one day after the 21st. If the full Moon is x days after the 21st, Easter will be $1 + x$ days after. And if the full Moon occurs x days after the 21st the Epact was x less than when it occurred on the 21st, or $23 - x$. Counting backwards, beginning with 21, fourteen days will include the 8th; so the 7th was the last day of the preceding Moon, and 7 from 30 leaves 23 as the Moon's age March 1st, *i. e.*, the Epact is 23. Let $P = 1 + x$, and $E = 23 - x$. Add the equations, transpose E, we get $P = 24 - E$. (1). January 1st is represented by A; March 1st, and 22d by D, which is the 4th letter of the alphabet. If then l stands for the days after the 21st to the full Moon (as l begins at 4) for x days after the 21st, we shall have $l = 4 + x$; and as before $E = 23 - x$. Add these equations and transpose E, we find $l = 27 - E$. (2). If then L denotes the number of the Dominical Letter (E = 5, F, 6) and l the number of the letter on the day of the full Moon ($L - l$) will give the number of days

from the full Moon to the next Sunday. If l is larger than L, add 7 to L and then subtract. In like manner, if l exceed 7, we throw out the sevens, for we are looking only for the next Sunday after l. Sometimes L equals l. Then if Q stands for the number of days after the 21st to Easter, we have $Q = P + (L - l)$. (3). Equations (1), (2), and (3) give Easter. Take two examples: first, find Easter for 1897.

The Golden Number for 1896 is 16; for 1897 it is 17. Subtract 1, multiply by 11, throw out the 30's, the remainder is 26, which is E, the Epact. The Epact being over 23, the full Moon occurs before the 21st, and we add 30 days. Equation (1) becomes $P = 54 - 26 = 28$; equation (2), $l = 57 - 26 = 31$, omitting the 7's we have 3. The Dominical Letter is $C = 3 = L$; then equation (3) gives $Q = 28 + (3 - 3) = 28$ days from March 21; or Easter is on April 18th, 1897. As another example, find Easter for 1929.

To the year add its fourth, and drop 1, the remainder is 2410; divide by 7, the remainder is 2, which denotes F, the 6th letter of the alphabet; then $L = 6$. Find the Golden Number by the rule; it is 11. Multiply 10 (1 less than the Golden Number) by 11, throw out the 30's, there remains 20; diminish this by 1, the remainder, 19, is the Epact for 1929. Then equation (1), $P = 24 - 19 = 5$; equation (2), $l = 27 - 19 = 8$; drop 7, 1 remains; $l = 1$. Equation (3), $Q = 5 + (6 - 1) = 10$. So Easter is 10 days after March 21st, that is, Easter is March 31, 1929, as you can easily find out by waiting to see.

The chief church festivals before Easter are these: Septuagesima, nine weeks; Sexagesima, eight weeks; Quinquagesima, or Shrove Sunday, seven weeks; the following Tuesday is Shrove Tuesday, and the next day is Ash Wednesday, the beginning of Lent; Palm Sunday, one week; Good Friday, the Friday before Easter. After Easter are these: Low Sunday, one week; Rogation Sunday, five weeks; the following Thursday is Ascension Day; Pentecost or Whitsunday, seven weeks; Trinity Sunday, eight weeks; the following Thursday is Corpus Christi.

If one curious in these matters will look into Webster's Unabridged Dictionary, edition of 1874, under the word *Sexagesima*, he will find the following definition of the word: "The second Sunday before Lent, the next to Shrove-Tuesday. . ."—and this latter clause is an error; Quinquagesima is "next to Shrove-Tuesday." And with this communication I dismiss the common Almanac.

THE GRAPHIC CONSTRUCTION OF ECLIPSES AND OCCULTATIONS.

II.—Solar Eclipses.*

WM. F. RIGGE, S. J.

For Popular Astronomy.

Solar and Lunar Disks. Besides the times of the phases, there are other data which we may gather from our diagram by fur-

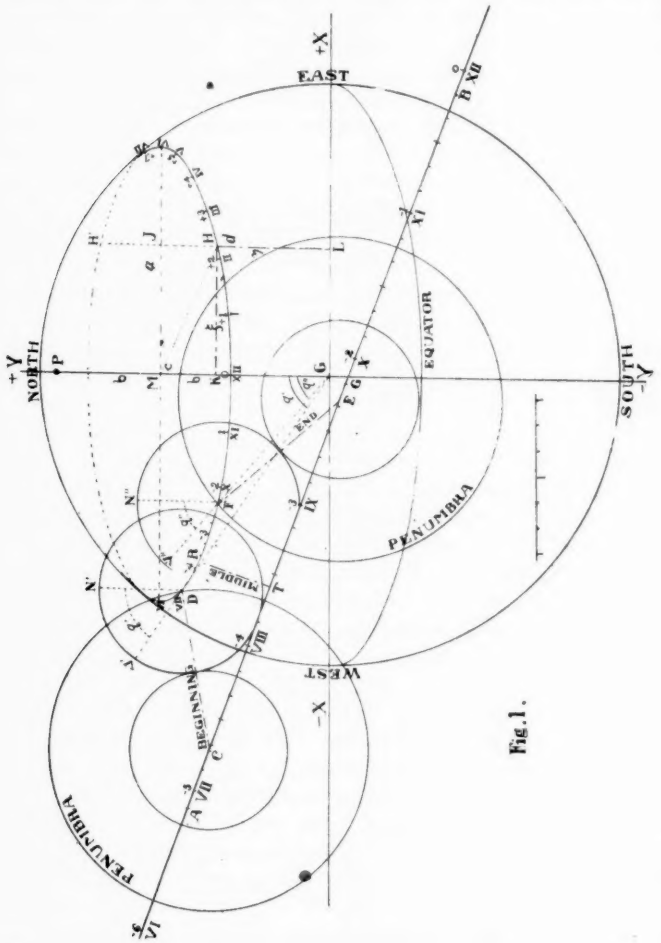


Fig. 1.

* Continued from page 255.

ther constructions. When St. Louis is on the circumference of the penumbra at the beginning and end of the eclipse, it sees the solar and lunar disks in contact. Hence we may replace each of the penumbral circles whose centers are at C and E, by two circles in contact with centers at C and D, and at E and F, the lunar disk being centered on the Moon's path to which it must be confined, and the solar disk being centered on the position of St. Louis. As the Earth and Moon when seen from the Sun, are seen at practically the same distance (their distance apart being only one four-hundredth of the Sun's distance), their apparent size is proportional to their true magnitudes. Astronomers have determined the relative radii of the Earth and Moon, and found the Moon's radius or semi-diameter to be 0.272 in comparison to the Earth's as 1.000. Hence, as we have called the radius of the Earth's disk unity, we draw the lunar disks at C and E with the radius 0.272. Now, as the sum of the radii of the Moon and Sun must equal 0.553, the radius of the penumbra, the radius of the Sun's disks at D and F must be equal to the remainder, that is, to 0.281. The Sun's semi-diameter is larger than the Moon's in this instance, and this is the reason why this eclipse is annular. The relative size of the disks varies in different eclipses on account of the variation of the penumbra caused by the varying distances of the Sun and Moon from the Earth.

Position Angles. In order that we may be able to observe the beginning of the eclipse with great accuracy, we must know at what part the moon will begin to obscure the solar disk. The position of this point must be measured from other points which are easily identified and located. The most important of these is the north point, which is the same for all the observers of the world. The north point is that point of the limb in which it is cut by a plane passing through the center and the north celestial pole. The line DN′ drawn through the center of the Sun parallel to the axis of Y, determines the north point of the disk. The angular distance of the point of contact from the north point, that is, the angle N′DC, will be found by measurement to be 101°. The Moon will, consequently, first encroach upon the Sun's disk at the point whose bearing upon the sun is N 101° W or S 79° W. The angle at the end of the eclipse, the angle N″FE, is less important, but it can very easily be measured. It will be found to be N 140° E or S 40° E. These angles may now be measured or estimated upon the Sun's disk in the sky, if we have any means, such as an equatorial mounting, of determining the cardinal points, but if not, we may with probably greater convenience measure them from

the Sun's vertex or uppermost point. This point is found with equal facility on the diagram. The line GD, which joins the center of the Earth's disk with the point D on the ellipse, will when produced cut the Sun's limb at the vertex V′, because this line, being the projection of a plane passing through St. Louis, the Earth's center and the Sun which is perpendicular to the drawing, is a vertical plane at St. Louis. Similarly V″, the extremity of a line from G through F, is the vertex at the end of the eclipse. This is an easy means of finding the Sun's vertex at any time, and as we see here, it is essentially a local quantity because it depends upon the latitude of the place and upon the declination and hour angle of the Sun, all three of which are necessary to locate any point on the ellipse. The angle between the north point and the vertex is called the parallactic angle; it is found by measurement to be $q' = \text{PGD} = \text{N}'\text{DV}' = 56°$ at the beginning, and $q'' = \text{PGF} = \text{N}''\text{FV}'' = 49°$ at the end of the eclipse. The angular distances of the points of contact from the vertices are then V′ 45° W, and V″ 189° E or V″ 171° W. We see also how the vertices on our diagram agree with those in the sky, they are between the N and W points of the limb in the morning or before meridian passage, and between the N and E points in the afternoon. Many more interesting facts will find their simple graphic explanation in this diagram, but they are somewhat beyond our purpose.

Sun's Altitude. There is, however, one of these facts so useful and so simple that we cannot forbear mentioning it. If we remember that D and G are the projections of points on a hemisphere, it will be easy to see that the straight line DG is the projected sine of the arc which joins these points on the Earth's surface. As this sine is parallel to the fundamental plane, it is not altered by the projection. Now the angle between D and G on the Earth's surface is the same as the angle between their zeniths. Hence, as St. Louis is at D, and the Sun is in the zenith of G, the line DG is the sine of the Sun's zenith distance at St. Louis at the beginning of the eclipse, or the cosine of the Sun's altitude. By measurement we find DG = 0.906, and looking in a table of natural sines, we find this to be the sine of 65°. Therefore the Sun's zenith distance at St. Louis at the beginning of the eclipse is 65°, or its altitude is 25°. Similarly, FG = 0.586 = sine 36°, that is, the Sun's zenith distance at the end of the eclipse is 36°, or its altitude 54°. We might find the Sun's altitude at the middle of the eclipse in like manner, that is, by measureing RG and determining the arc of which it is the cosine, as also the altitude at noon or at any time during that day.

The Eclipse as seen in the Sky. We shall find it very convenient when about to observe the eclipse, to have at hand another and smaller diagram, which will represent the eclipse exactly as we expect to see it in the sky, and contain in brief all the information that may be of service to us at that time. Such a diagram will be found in Fig. III, which either in the drawing itself, or in the data below it, or in both, contains all the elements of the eclipse pertaining to St. Louis.

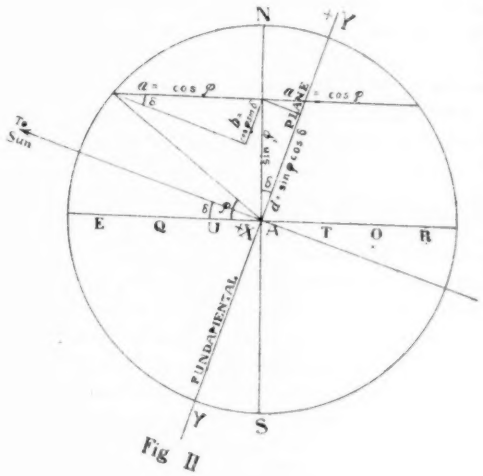


Fig II

We begin by drawing a circle with any radius to represent the Sun. For convenience in transferring quantities from our other diagram, we shall call its length 0.281. We mark on the circle the cardinal points, taking care to place W on the right hand, because this is the case when we look at the sky. The center of the Sun in the diagram is indicated by the three letters D, R, F, in order that the reader may see with greater facility how the quantities from Fig. I are transferred to Fig. III. We draw the angle NDC in Fig. III equal to the angle N′DC in Fig. I, remembering to place C on the right side instead of on the left as it is in Fig. I, for the reason just given. We also lay off the angle NFE = N″FE in Fig. I. Then laying off the points C and E at the distance 0.553 (the radius of the penumbra) from the Sun's center, that is, making DC and FE the same as they are in Fig. I, we draw for the Moon's disk two circles to the radius 0.272 with their centers at C and E. These circles will then represent the solar and lunar disks in contact at the beginning and end of the eclipse. We next draw the angle NRT equal to N‴RT on Fig. I. This letter N‴ must be imagined to be in Fig. I on a line drawn

NOTE.—The above figure is referred to on pages 251 and 252 of the January number.

through R parallel to the axis of Y. We have omitted this line, as well as the penumbral circle and the solar and lunar disks with their centers at R and T, because Fig. I is sufficiently crowded. Laying off RT in Fig. III equal to RT in Fig. I, (or of a proportionate length, if the scale of these figures is different) and drawing the lunar disk with its center at T (and radius 0.272) we have the Moon's position at the middle of the eclipse when it has entered farthest upon the Sun. We may now connect the points C, T, E, by an easy curve in order to show the apparent path of the Moon's center. A more accurate determination of the positions of points intermediate between C and T, and between T and E, may be made from Fig. I for every ten minutes in the same way as these three points have been found, and the exact position of the Moon with regard to the Sun, as also the exact representation of the progress of the eclipse and of the obscuration determined, if we feel so inclined, but this is hardly necessary. The vertices V′ and V″ are located by making the angles NDV′ and NFV″ equal to N′DV′ and N″FV″ on Fig. I. At the time of the eclipse we then need but hold Fig. III in such a way that V′ is the uppermost point on the solar disk at the beginning, and V″ the uppermost point at the end of the eclipse, in order to have an exact picture of the eclipse on paper.

Magnitude. All but one of the quantities printed below Fig. III are those we have already determined in the course of this article. The one exception is the magnitude of the eclipse, which is found more conveniently from Fig. III, although it might also be found from Fig. I. The magnitude of the eclipse is the fraction of Sun's diameter obscured at the middle of the eclipse. At this moment the Moon's center is at T on Fig. III. We produce the line RT if necessary until it cuts the nearest points of the limbs of the Sun and Moon. This point on the Sun's limb is U, but as the one on the Moon's limb is so near the Sun's center that is impossible to crowd in another letter near D, F, R, we shall imagine a letter to be placed there on the intersection of RT produced and the Moon's limb, and call that point D′. Measuring DU′, we find it to be 0.252. Dividing this obscured part of the Sun's diameter by the whole diameter, that is, by twice 0.281, we see that 54 hundredths of it are obscured, that is, 0.54 is the magnitude of the eclipse for St. Louis, the Sun's diameter being taken as unity.

Accuracy. If the construction has been made carefully, we shall find our results as a general thing to be within a minute of the truth. Thus in the present instance, subsequent rigorous calculation gave corrections of $+ 1^m 9^s.1$ and $- 0^m 24^s.0$ to the times of

beginning and ending of the eclipse as determined by the graphic construction of Fig. I. This error may often be less than that of our time pieces. Besides, we have, as in the case of a lunar eclipse in our last article, supposed all the elements of the eclipse to be constant during the time it takes the penumbra to cross the Earth's disk. This is not rigorously true. In addition we have practically ignored the Sun's parallax. Our greatest sin has probably consisted in using the geographic latitude $\varphi = 38° 38'$, instead of the geocentric, $\varphi' = 38° 27'$. The difference between the two never amounts to one fifth of a degree, and will scarcely be discernible on our diagram, cos φ being 0.781 and cos φ', 0.783. Still, if this quantity is to be used frequently, we might as well use the geocentric latitude. This is given in many text books and in the Ephemeris on page 509. Multiplying sin φ' and cos φ' by ρ, the local terrestrial radius, in order to apply a correction for the spheroidal shape of the Earth, is entirely unnecessary in our work, because the smallest value of ρ, at the poles of the Earth, is 0.997 instead of 1.000, and at St. Louis it is 0.999.

Construction for another place. In order to construct the present solar eclipse for another place, we must make two changes, one dependent upon our change of latitude and the other upon our change of longitude. The change of latitude will alter all the constants of the ellipse, a, b, d, so that an entirely new ellipse

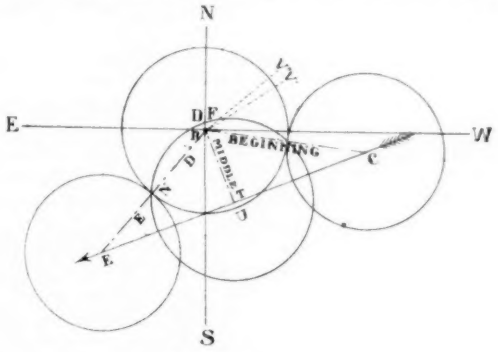


Fig. III.

must be plotted. The change of longitude will only change the reading of the point G on the Moon's path. All other quantities are the same. Of course, on account of our making the two changes mentioned, the times of the phases, the magnitude, posi-

tion angles and the like, will all be different from those we have found for St. Louis.

GEORGETOWN COLLEGE OBSERVATORY,
Washington, D. C.

SOLAR ECLIPSE, JULY 29, 1897.

FOR ST. LOUIS, MO.

Beginning.			End.	
h	m		h	m
— 4	43	Sun's Hour Angle.	— 2	17
7	17 A. M.	Local Apparent Time.	9	43 A. M.
7	23 A. M	Local Mean Time.	9	49 A. M.
7	24 A. M.	Central Time.	9	50 A. M.
	25°	Sun's Altitude.		54°

N 101° W	Points of Contact.	N 140° E
S 79 W		S 40 E
V 45 W		V 171 W
N 56 W	Position of Vertex.	N 49 W

Middle of the Eclipse $8^h\ 32^m$ A. M. Central Time.
Magnitude 0.54

PRACTICAL SUGGESTIONS.

The Constant of Gravitation —It does not seem to the writer that the objection raised by Professor Young in the November issue of ASTRONOMY, with reference to the constant "k," is well taken.

The fundamental relations existing between the attractive force between two masses M_1 and M_2 separated by a given distance, d, is

$$\text{Force} \propto^* \frac{M_1 \times M_2}{d^2}$$

$$\text{Force} = k \frac{M_1 \times M_2}{d^2}$$

The question at issue is: What is the real signification of the constant k in the second equation given. The writer, in the article referred to insisted that k is simply the numerical constant required to convert proportionality into equality. Boys evidently takes the same view of the significance of this constant, for he says (see page 129) "Newton showed that to complete his law and to put in the numerical constant that would convert his proportion into an equality, two methods are available; we may either make observations of the disturbance of the Earth's gravitation by the action of the isolated parts of it, or we may make an artificial planet of our own and find the attraction which it exerts."

In the present instance, the writer compared two well defined masses, viz:—a mass of 1 gramme and the entire mass of the Earth concentrated at its center of gravity. This conception enables us to find the numerical value of k expressed in static units, if we assume 1 gramme as the unit of mass and if we know with sufficient accuracy the length of the Earth's radius and the density of the Earth.

In the equation

$$k = \frac{Fd^2}{M_1 \times M_2}$$

* The symbol $\propto$ means "varies as."

k is really an involved function of the despised "g" through the unit of force adopted.

Now the objection may properly be raised that the dimensions and the density of the Earth are not known with sufficient accuracy to give an exact value of k. Probably the language of the previous note was not sufficiently explicit on this point, but it may be stated here that the statement objected to by Professor Young is strictly true, if this method of finding the numerical value of k is adopted. I suspect that it would be found that all experiments made previous to those by Boys would give a far less accurate value of this constant than the value determined in the manner indicated above. Boys' method gives the force of attraction between the known masses expressed in absolute units, only when the modulus of torsion is exactly known. In the previous work of Cavendish the uncertainty in regard to the value of this factor of the observed deflections must have been very great. Boys, in one of the most brilliant series of experiments of modern times, has reduced this uncertainty to a minimum by the use of quartz fibers in making the suspensions of given masses of matter of known dimensions and of known density, but there must remain some uncertainty in the value of the torsional factor.

My object in writing the note to which reference has been made, was to remove some of the haziness with which the mind of the average student is affected in regard to this constant. Boys, in his article, magnifies the office of k, or G, according to his notation, and intimates that g is of the earth, earthy. He speaks of it as "arbitrary, incidental and vexatious." The fact is, the two constants rest upon precisely the same footing in one respect, viz: that they both require for their exact determination, at least two elements which must be determined experimentally. If k is determined from the mass of the Earth, this uncertainty results from our lack of the knowledge of the exact mass of the Earth with which the mass of one gramme is compared; or if the torsional method is employed, it rests upon the uncertainty, in our present knowledge of the modulus of torsion, even if the observed deflections are considered as having been made without error.

In order that one may form an estimate of the effect of an assumed error in the radius of the Earth, we will compute the values of k for the two values $r_1 = 3959$ miles and $r_2 = 3859$ miles, giving the details of the computation in both cases.

We have:

The attraction of the Earth having a mass M_1 supposed to be concentrated at its center of gravity, upon a mass of 1 gramme (expressed by m) *varies as* $\frac{M}{r^2}$ in which r represents the mean radius of the Earth.

We have then:

The attraction upon 1 gramme $\propto \frac{M}{r^2}$ static units. Assuming the attraction of 1 gramme to be 981 dynes, we have:

$$981 \text{ dynes} \propto \frac{M}{r^2} \text{ static units.}$$

Converting proportionality into equality, we have:

$$981 \text{ dynes} = \frac{kM}{r^2} \text{ static units.}$$

Hence $$1 \text{ dyne} = \frac{k\mathrm{M}}{981r^2} \text{ static units};$$

and $$1 \text{ static unit} = \frac{981}{k\dfrac{\mathrm{M}}{r^2}} \text{ dynes} = \frac{981r^2}{k\mathrm{M}} \text{ dynes}.$$

For the numerical computation we have:

$$r_1 = 3959 \times 5280 \times 12 \times 2.54 = 637140000 \text{ cms.}$$
$$r_2 = 3859 \times 5280 \times 12 \times 2.54 = 621050000 \text{ cms.}$$
$$\mathrm{M} = \tfrac{1}{4} \text{ vol.} \times \text{density of the Earth} = \frac{4}{3}\pi r^3 \times 5.6$$
$$\log \mathrm{M}_1 = 27.7829823$$
$$\log \mathrm{M}_2 = 27.7496574$$
$$\frac{\mathrm{M}_1}{r^2} = 14945000000$$
$$\frac{\mathrm{M}_2}{r^2} = 14568000000$$
$$k_1 = 0.0000000656$$
$$k_2 = 0.0000000673$$

It is to be noted that it cannot be assumed that because the value obtained by Boys, falls between the two values of k thus determined, the radius of the Earth must fall between the values 3859 and 3959 miles.

It is also to be noted that k may be defined to be the abstract number which, used as a factor of one term, will reduce it to the same denomination as the other term.

This use of k as a factor to convert proportionality into equality seems to the writer to be its legitimate function. It is however, frequently not recognized even when it necessarily exists in a given relation. In some problems its numerical value can be easily obtained from observed data. In other cases the value must be considered as equal to unity, in order to obtain an equation which will express equality. Thus, the relation between units of force, mass and of acceleration is almost always incorrectly written

$$f = ma$$

It should always be written $f \propto ma$

and $f = kma$

So also we should write Momentum $\propto \mathrm{M}v = k\mathrm{M}v$.

In the former case the numerical value of k can be determined experimentally. In the latter it cannot be readily found; in this case we must understand that $k = 1$, if we write

$$\text{Momentum} = \mathrm{M}v.$$

I have said that the value of k in the equation,

$$f = kma$$

can be determined from observation.

Balance a beam upon a knife-edge fulcrum. Place a mass upon the beam, whose weight is one pound, at a given distance from the fulcrum. Let fall from a height of 16.08 feet, a ball of lead, weighing a little over half an ounce from a point such that the ball, in falling, will strike the beam on the side opposite the weight, and at exactly the same distatce from the fulcrum as the weight.

It will be seen that the momentum acquired by the falling ball will raise the

pound-weight a slight distance. It will be found that if the weight is much less than half an ounce the blow will not raise the weight. By successive trials, cutting off a little of the lead after each operation, we find that the exact weight required to give a perceptible elevation to the pound-weight is $\frac{1}{32.16}$ lb. Of course this value can be obtained analytically for the definition of a unit of force.

The unit of force will then be found to be $\frac{1}{32.16}$ lb., the weight of the pound-mass being assumed as 1 lb.

Hence

$$\frac{1}{32.16} = kma$$

If m = a unit of mass, and a = a unit of acceleration; then

$$\frac{1}{32.16} = k$$

Hence for any mass M, and for any force F, we have, representing the acceleration of gravity by g

$$\text{"vexations } g\text{"} = \frac{32.16\ \mathrm{F}}{\mathrm{M}}$$

It may be desirable to give another illustration: When the wire of a sonometer is plucked, the number of vibrations n, per second, in the sound produced, varies (1) as the square root of the tension T produced by the weight Wg, by which the wire is stretched: (2) it also varies inversely as the square root of the area of the cross-section of the wire, or as the $\sqrt{\frac{1}{\pi r^2}}$, (3) n also varies inversely as the length of the wire, and again, (4) inversely as the square root of the density of the wire.

We thus have

$$n \propto \frac{1}{l}\sqrt{\frac{\mathrm{W}g}{\pi r^2 d}}$$

$$n \propto \frac{1}{rl}\sqrt{\frac{\mathrm{W}g}{\pi d}}$$

$$n = k \times \frac{1}{rl}\sqrt{\frac{\mathrm{W}g}{\pi d}}$$

In this case, the constant k is made up of two factors, one of which is the abstract number $\frac{1}{2}$; for, when we pass from proportionality to equality we must write $2l$ for l, since the length of the wire plucked is only one-half of the length of a wave. The other factor expresses the number of octaves above the lowest fundamental note of which the wire is capable. Representing this number only by k we have:

$$n = k \times \frac{1}{2rl}\sqrt{\frac{\mathrm{W}g}{\pi d}}$$

All of these relations can be verified in a single experiment. Let us assume:

$$W = 5000 \text{ grammes.}$$
$$g = 981 \text{ dynes.}$$
$$n = 256$$
$$r = 0.01 \text{ cm.}$$
$$d = 8.7$$

We have then

$$l = \frac{1}{2 \times 256 \times 0.01} \sqrt{\frac{5000 \times 981}{3.1416 \times 8.7}}$$

Measuring off the length thus computed, upon the sonometer, and plucking the wire at the same time that a C tuning fork is sounded, we can assume that all the relations given by the equation hold true if the two sounds produce harmony. On the other hand, if we measure off a value of l which will produce harmony, then "vexatious g" will be the only unknown quantity in the equation.

It will be found that for the higher octaves a sound produced by a length of the wire l can be distinguished from a sound corresponding to a length $l \pm$ one millimeter with the greatest ease.

In a series of observatians made by the writer for the determination of "vexatious g" the value came out 978 dynes, with a probable error of 15 units for a single observation.

I have given these illustrations for two reasons. First, in order to make the nature of the constant k clear to the student who meets this term in his reading and secondly, in order to illustrate the fact that while there may be two or even several ways of obtaining the value of a physical constant, each being theoretically correct, the inherent precision of one may be vastly greater than that of the other. For example, no one would think of employing either of the methods described above for a precise determination of g, but they are both theoretically correct.

With regard to the question under consideration, the methods given in the article referred to, and the method employed by Boys are both strictly correct, (if Professor Young will allow the assertion). In the present case the real question is, which method has the greater inherent precision. Two masses are suspended to swing side by side, at a known distance apart. One of these masses is the definite quantity which we call one gramme. We will assume that the other, the concentrated mass of the Earth, has the same volume, but a comparatively greater density. If we can determine the law of attraction which exists between these two particular masses, we can determine the mass of any other body of matter, in the simplest possible manner, viz., by assuming the mass of the Earth as the unit, in the same manner as we, in the first instance, assumed the gramme as the unit. All the data we need for this purpose, is the distance of the assumed body, its mean radius in any unit which we may have adopted, and its density. In other words, we start with the gramme and proceed to the vastly greater mass, the Earth. Then, assuming the mass of the Earth as the unit we can proceed to the determination of a different mass situated at any point in the universe.

In view of what has been said the writer cannot agree with Professor Boys that k "represents a mighty principle under the influence of which every star, planet and satellite in the universe pursued its allotted course." Indeed, the sentences which follow this quotation have the appearance of an attempt at fine writing. On the other hand, while g is not constant for all points upon the surface of the Earth, it certainly is constant for any given point, for a given mass will always have the same acceleration at this point. g is therefore, a real constant of nature. It is neither arbitrary, incidental, nor vexatious, and it is certainly *not* accidental.

The question may be fairly raised whether a numerical constant can be regarded as representing *any* mighty principle in nature until we know what gravitation really is, we can hardly do more than study the consequences which flow from the application of this universal force to matter as it exists. Since the force which expresses the mutual attraction of two masses of matter is a function of those masses and of the distance by which they are separated, the force itself cannot be represented by a numerical constant. If the conception offered by Boys is a true one, we may have one and the same "mighty principle" in nature represented by two different constants, for no one will claim that the Gaussian constant "k," is not as truly a constant of nature as the constant "k" under consideration.

If Professor Boys has so low an opinion of "arbitrary incidental vexatious and accidental "g," he ought to be consistent and withdraw it from the numerical value of "k" which he obtains and express his result in some other than C. G. S. units.

Notwithstanding the objections offered, it still appears to the writer proper to speak of "k" as a constant of mass and to speak of the Gaussian constant "k" as a constant of planetary motion. WILLIAM A. ROGERS.

Colby University, Jan. 1, 1896.

THE PLANETS AND THE CONSTELLATIONS FOR FEBRUARY.

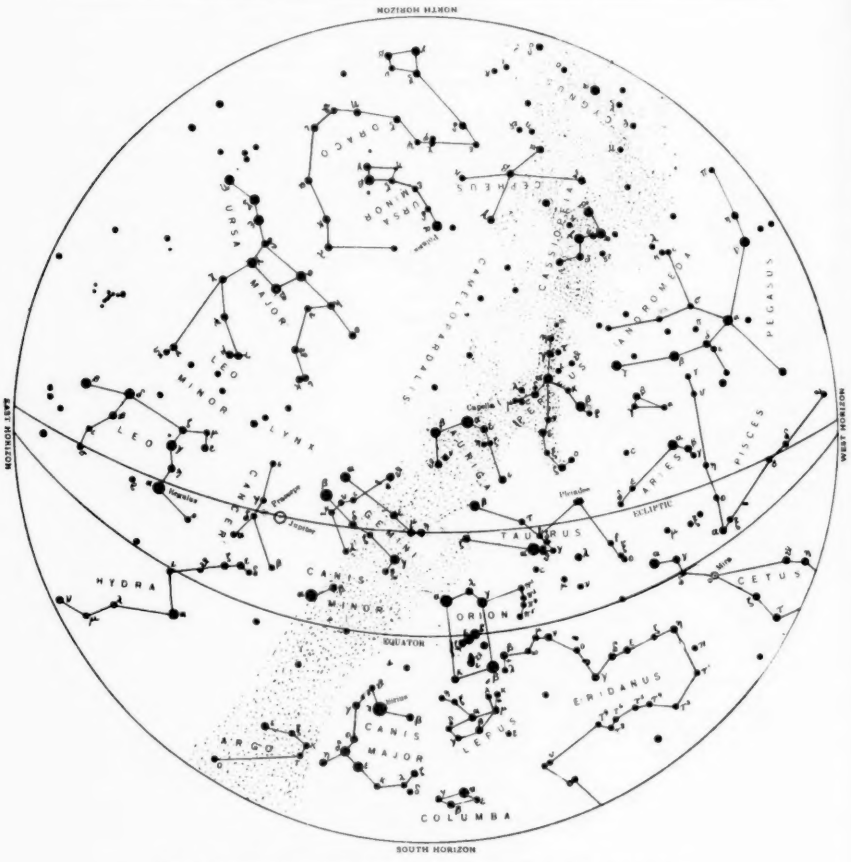
Mercury comes to inferior conjunction with the Sun Feb. 7 at midnight, Central Standard time, when the planet will be nearly 4° north of the Sun. During the last days of the month Mercury will be visible in the morning, about an hour before sunrise. The observer should look toward the southeast horizon, about 10° east from Venus.

Venus is still the bright "morning star," seen toward the southeast by early risers. She is moving away from us, so that her light diminishes in the ratio of the numbers 82 to 67, although her phase increases from 0.75 to 0.83. She passes by *Mars*, as seen from the Earth, February 9 at 2^h P. M., when Mars will be 1° 38′ south of Venus. Although the two planets appear so close together, the reader will remember that really the one is at a great distance beyond the other, Mars at this date being 1.8 times as far as Venus from the Earth.

Jupiter is seen toward the east in the early evening and reaches the meridian, high up toward the zenith, at half past eleven. At the end of February the meridian passage will occur as early as half past nine, so that the planet is in very favorable position for observation during this month. The great red spot will be near the central meridian of Jupiter's disc at the following times:

Feb.	1	8 P. M.	Feb.	11	6 P. M.	Feb.	21	5 P. M.
	2	4 "		12	midn.		22	10 "
	3	10 "		13	8 P. M.		23	6 "
	4	6 "		14	4 "		24	midn.
	5	11 "		15	10 "		25	8 P. M.
	6	7 "		16	6 "		27	2 A. M.
	8	1 A. M.		17	11 "		27	10 P. M.
	8	9 P. M.		18	7 "		28	5 "
	9	5 "		20	1 A. M.		29	11 "
	10	10 "		20	9 P. M.			

This spot is not conspicuous now as it was 15 years ago, and were it not for the notch in the adjacent red belt, it would be very difficult to recognize the spot.



THE CONSTELLATIONS FOR FEBRUARY 1 AT 9 O'CLOCK P. M.

In the last number of *Knowledge* (Jan. 1, 1896) there is a very interesting article on the Great Red Spot on Jupiter, by Mr. E. W. Maunder, in which the writer claims that the red spot is at a lower level than the white cloudy envelope around it and therefore its permanence of outline is due to the permanence of the cloud forms. The explanation of its permanence is, however, not helped by this suggestion.

Saturn and *Uranus* are both morning planets, and are to be looked for toward the south at 6 o'clock a. m. Saturn is easily recognized, by his brightnes, and golden yellow color, among the stars of Libra. Uranus must be found with a telescope in right ascension $15^h\ 28^m$, declination south $18°\ 36'$. Saturn will be at quadrature, 90° west from the sun, Feb. 7. Uranus will be in a corresponding position Feb. 13.

Neptune is in the opposite part of the heavens, and may be found in the evening in Taurus, in right ascension $4^h\ 57^m$ and declination $21°\ 12'$ north.

Jupiter's Satellites for February.

Central Standard Time.

Phases of the Eclipses of the Satellites for an Inverting Telescope.

I. r

III. r

II. r

IV. d r

Configuration 10h 30m p. m. for an Inverting Telescope.

Day.	West.	East.
1	4· 2· 1· ○ 3·	
2	·4 3· ○ 1·	·2●
3	·4 ·3 ·1 ○ 2·	
4	·4 ·3 2· ○ ·1	
5	·2 ·4 ·1 ○ ·3	
6	○1· ○ ·4 ·2 ·3	
7	○ ·12· 3··4	
8	2· 1· ○ 3· ·4	
9	3· ·2 ○ ·1	·4
10	3· ·1 ○ 2·	4·
11	·3 2· ○ 1·	4·
12	·2 ·1 ○ ·3 4·	
13	○1 ○ ·2 4· ·3	
14	4· ○ ·1 2· 3·	
15	4· 2· 1· ○ 3·	
16	4· 3· ·2 ○ ·1	
17	4· 3· 1· ○ ·2	
18	○2·4· ·3 ○ 1·	
19	·4 ·2 ·1 ○	·3●
20	·4 ○ 1· ·2 ·3	
21	·4 ○ 2· 3·	·1●
22	2· 1· ·4 ○ 3·	
23	3· ·2 ○ ·1 ·4	
24	3· 1· ○ ·2 ·4	
25	○2· ·3 ○ 1· ·4	
26	·2 ·1 ·3 ○	·4
27	○ 1· ·2 ·3 4·	
28	·1 ○ 2· 3· 4·	
29	2· 1·○ 3· 4·	

Phenomena of Jupiter's Satellites.

Central Time.

Feb.	1	1	17	P. M.	I	Tr. In.
		1	29	"	I	Sh. In.
		3	37	"	I	Tr. Eg.
		3	49	"	I	Sh. Eg.
		11	23	"	III*	Tr. In.
	2	12	17	A. M.	III*	Sh. In.
		3	2	"	III*	Tr. Eg.
		3	55	"	III*	Sh. Eg.
		10	24	"	I	Oc. Dis.
		12	55	P. M.	I	Ec. Re.
		9	8	"	II*	Oc. Dis.
	3	12	30	A. M.	II*	Ec. Re.
		7	43	"	I	Tr. In.
		7	58	"	I	Sh. In.
		10	3	"	I	Tr. Eg.
		10	18	"	I	Sh. Eg.
	4	4	50	"	I*	Oc. Dis.
		7	24	"	I	Ec. Re.
		3	12	P. M.	II	Tr. In.
		3	47	"	II	Sh. In
		6	7	"	II	Tr. Eg.
		6	42	"	II*	Sh. Eg.
	5	2	9	A. M.	I*	Tr. In.
		2	26	"	I*	Sh. In.
		4	29	"	I*	Tr. Eg.
		4	46	"	I*	Sh. Eg.
		12	53	P. M.	III	Oc. Dis.
		5	42	"	III*	Ec. Re.
		11	16	"	I*	Oc. Dis.
	6	1	52	A. M.	I*	Ec. Re.
		10	15	"	II	Oc. Dis.
		10	30	"	IV	Oc. Dis.
		1	48	P. M.	II	Ec. Re.
		6	17	"	IV*	Ec. Re.
		8	35	"	I*	Tr. In.
		8	55	"	I*	Sh. In.
		10	55	"	I*	Tr. Eg.
		11	15	"	I*	Sh. Eg.
	7	5	41	"	I*	Oc. Dis.
		8	21	"	I*	Ec. Re.
	8	4	20	A. M.	II	Tr. In.
		5	5	"	II	Sh. In.
		7	15	"	II	Tr. Eg.
		8	0	"	II	Sh. Eg.
		3	1	P. M.	I	Tr. In.
		3	23	"	I	Sh. In.
		5	21	"	I*	Tr. Eg.
		5	43	"	I*	Sh. Eg.
	9	2	40	A. M.	III*	Tr. In.
		4	16	"	III*	Sh. In.
		6	19	"	III	Tr. Eg.
		9	55	"	III	Sh. Eg.
		12	8	P. M.	I	Oc. Dis.
		2	50	"	I	Ec. Re.
		11	23	"	II*	Oc. Dis.
	10	3	6	A. M.	II*	Ec. Re.
		9	27	"	I	Tr. In.
		9	52	"	I	Sh. In.
		11	47	"	I	Tr. Eg.
		12	12	P. M.	I	Sh. Eg.
	11	6	34	A. M.	I	Oc. Dis.
		9	18	"	I	Ec. Re.
		5	28	P. M.	II*	Tr. In.
		6	23	"	II*	Sh. In.
		8	23	"	II*	Tr. Eg.
		9	18	"	II*	Sh. Eg.
	12	3	53	A. M.	I*	Tr. In.
		4	21	"	I	Sh. In.
		6	13	"	I	Tr. Eg.
		6	41	"	I	Sh. Eg.
		4	11	P. M.	III	Oc. Dis.
		9	41	"	III*	Ec. Re.
	13	1	0	A. M.	I*	Oc. Dis.
		3	47	"	I*	Ec. Re.
		12	31	P. M.	II	Oc. Dis.
		4	23	"	II	Ec. Re.
		10	20	"	I*	Tr. In.
		10	50	"	I*	Sh. In.
	14	12	40	A. M.	I*	Tr. Eg.
		1	10	"	I*	Sh. Eg.
		5	2	P. M.	IV	Tr. In.
		7	26	"	I*	Oc. Dis.
		9	43	"	IV*	Tr. Eg.
		10	0	"	IV*	Sh. In.
		10	16	"	I*	Ec. Re.
	15	2	40	A. M.	IV*	Sh. Eg.
		6	36	"	II	Tr. In.
		7	41	"	II	Sh. In.
		9	31	"	II	Tr. Eg.
		10	36	"	II	Sh. Eg.
		4	46	P. M.	I	Tr. In.
		5	18	"	I*	Sh. In.
		7	6	"	I*	Tr. Eg.
		7	38	"	I*	Sh. Eg.
	16	6	0	A. M.	III	Tr. In.
		8	15	"	III	Sh. In.
		9	39	"	III	Tr. Eg.
		11	54	"	III	Sh. Eg.
		1	53	P. M.	I	Oc. Dis.
		4	44	"	I	Ec. Re.
	17	1	39	A. M.	II*	Oc. Dis.
		5	41	"	II	Ec. Re.
		11	12	"	I	Tr. In.
		11	47	"	I	Sh. In.
		1	32	P. M.	I	Tr. Eg.
		2	7	"	I	Sh. Eg.
	18	8	19	A. M.	I	Oc. Dis.
		11	13	"	I	Ec. Re.
		7	45	P. M.	II*	Tr In.
		8	59	"	II*	Sh. In.
		10	40	"	II*	Tr. Eg.
		11	54	"	II*	Sh. Eg.
	19	5	38	A. M.	I	Tr. In.
		6	15	"	I	Sh. In.
		7	58	"	I	Tr. Eg.
		8	35	"	I	Sh. Eg.
		7	32	P. M.	III*	Oc Dis.
	20	1	41	A. M.	III*	Ec. Re.
		2	45	"	I*	Oc. Dis.
		5	42	"	I	Ec. Re.

20	2	48 P. M.	II	Oc.	Dis.
	6	59 "	II*	Ec.	Re.
21	12	4 A. M.	I*	Tr.	In.
	12	44 "	I*	Sh.	In.
	2	24 "	I*	Tr.	Eg.
	3	4 "	I*	Sh.	Eg.
	9	12 P. M.	I*	Oc.	Dis.
22	12	11 A. M.	I*	Ec.	Re.
	8	54 "	II	Tr.	In.
	10	17 "	II	Sh.	In.
	11	49 "	II	Tr.	Eg.
	1	12 P. M.	II	Sh.	Eg.
	6	31 "	I*	Tr.	In.
	7	13 "	I*	Sh.	In.
	8	51 "	I*	Tr.	Eg.
	9	33 "	I*	Sh.	Eg.
23	12	59 A. M.	IV*	Oc.	Dis.
	5	39 "	IV	Oc.	Re.
	7	47 "	IV	Ec.	Dis.
	9	22 "	III	Tr.	In.
	12	14 P. M.	III	Sh.	In.
	12	22 "	IV	Ec.	Re.
	1	1 "	III	Tr.	Eg.
	3	38 "	I	Oc.	Dis.
	3	53 "	III	Sh.	Eg.
	6	39 "	I*	Ec.	Re.
24	3	57 A. M.	II	Oc.	Dis.
	8	16 "	II	Ec.	Re.
	12	58 P. M.	I	Tr.	In.
	1	42 "	I	Sh.	In.
	3	18 "	I	Tr.	Eg.
24	4	2 "	I	Sh.	Eg.
25	10	5 A. M.	I	Oc.	Dis.
	1	8 P. M.	I	Ec.	Re.
	10	4 "	II*	Tr.	In.
	11	35 "	II	Sh.	In.
26	12	59 A. M.	II*	Tr.	Eg.
	2	30 "	II*	Sh.	Eg.
	7	24 "	I	Tr.	In.
	8	10 "	I	Sh.	In.
	9	44 "	I	Tr.	Eg.
	10	30 "	I	Sh.	Eg.
	10	57 P. M.	III*	Oc.	Dis.
27	4	32 A. M.	I	Oc.	Dis.
	5	41 "	III	Ec.	Re.
	7	37 "	I	Ec.	Re.
	5	7 P. M.	II	Oc.	Dis.
	9	34 "	II*	Ec.	Re.
28	1	50 A. M.	I*	Tr.	In.
	2	39 "	I*	Sh.	In.
	4	10 "	I	Tr.	Eg.
	4	59 "	I	Sh.	Eg.
	10	58 P. M.	I*	Oc.	Dis.
29	2	5 A. M.	I*	Ec.	Re.
	11	15 "	II	Tr.	In.
	12	53 P. M.	II	Sh.	In.
	2	10 "	II	Tr.	Eg.
	3	48 "	II	Sh.	Eg.
	8	17 "	I*	Tr.	In.
	9	7 "	I*	Tr.	In.
	10	37 "	I*	Tr.	Eg.
	11	27 "	I*	Sh.	Eg.

Ephemeris of Short-Period Variables.

FEBRUARY, 1896.

[Greenwich Mean Time].

2279 T Monocerotis.

Max.	Min.
13.35	5.42

2509 ζ Geminorum.

5.36	
15.52	10.50
25.67	20.66

4805 W Virginis.

16.33	8 13
	25.40

6758 β Lyræ.

	5.00
	17.92

6404 Y Ophiuchi.

1.36	12.24
18.49	29.37

6984 U Aquilæ.

Max.	Min.
2.03	
9.05	6.80
16.08	13.83
23.11	20.86

7124 η Aquilæ.

4.64	2.26
11.82	9.44
18.99	16.61
26.17	23.79

7149 S Sagittæ.

3.19	
11.57	8.17
19.95	16.55
28.33	24.93

7437 X Cygni.

Max.	Min.
9.46	2.66
25.85	19.05

7483 T Vulpeculæ.

3.10	1.80
7.54	6.24
11.98	10.68
16.41	15.11
20.85	19.55
25.28	23.98
29.72	28.42

8073 δ Cephei.

2.43	
7.79	6.20
13.16	11 57
18.53	16.94
23.89	22.30
29.26	27.67

Occultations Visible at Washington.

Date 1896.	Star's Name.	Magnitude.	IMMERSION Washington M. T.	IMMERSION Angle from N pt.	EMERSION Washington M. T.	EMERSION Angle from N pt.	Duration.
			h m	°	h m	°	h m
Feb. 15	14 Piscium*	5.9	8 2	86	8 54	220	0 52
19	26 Arietis	6.0	9 8	50	10 10	179	1 2
21	p Tauri	6.0	6 52	33	7 56	302	1 4
22	W. iv, 1421	6.0	7 23	109	8 45	246	1 22
24	A Geminorum	5.7	14 6	175	14 30	226	0 24
25	η Cancri*	5.4	17 32	152	18 8	251	0 36
26	80 Cancri	6.8	7 1	86	8 4	320	1 3
27	α Leonis	1.3	6 49	82	7 40	331	0 51
27	44 Leonis	6.0	17 5	151	17 52	270	0 47
28	75 Leonis	5.7	15 58	174	16 40	257	0 42
28	79 Leonis*	6.0	18 53	87	19 36	328	0 43

* Below the horizon at Washington.

Minima of the Variable Stars of the Algol Type.

[Given to the nearest hour in Central Standard Time.]

U CEPHEI. 1896.

	d	h
Mar.	4	16
	9	15
	14	15
	19	15
	24	14
	29	14

ALGOL.

	d	h
Mar.	8	17
	11	14
	14	10

ALGOL CONT. 1896.

d	h
17	7
28	18

λ TAURI.

	d	h
Mar.	5	9
	9	8
	13	7

R CANIS MAJ.

	d	h
Mar.	5	9
	6	12
	13	8
	14	11
	22	10

S CANCRI. 1896.

	d	h
Mar.	5	9
	24	8

δ LIBRÆ.

	d	h
Mar.	7	12
	14	11
	21	11
	28	10

U CORONÆ.

	d	h
Mar.	4	10
	11	7
	21	16
	25	13

U OPHIUCHI. 1896.

	d	h
Mar.	3	14
	8	15
	13	16
	14	12
	18	16
	19	12
	23	17
	24	13
	29	14

Continued from Hartwig's ephemeris in the *Vierteljahrsschrift*.

Marengo, Ill., 1896, Jan. 10. J. A. PARKHURST.

An Annular Eclipse of the Sun will take place February 13, 1896. It will be visible in the Antarctic regions. The central line of the annulus passes within 6° of the south pole and ends south of Africa. The eclipse will be partial along the southeast coast of South America and in the southwestern portion of south Africa.

ELEMENTS OF THE ECLIPSE.

Greenwich mean time of conjunction in right ascension, February 13, $3^h\ 32^m\ 25^s.9$

Sun and Moon's R. A.	$21^h\ 47^m\ 8^s.01$	Hourly motions	$9^s.79$ and $113^s.38$
Sun's declination	13° 22′ 12″.3 S.	Hourly motion	0′ 50″.5 N.
Moon's declination	14 17 33 .2 S.	Hourly motion	12 35 .0 N.
Sun's equa. hor. par.	8 .9	Sun's true semidiam.	16 11 .7
Moon's equa. hor. par.	54 29 .3	Moon's true semidiam.	14 50 .1

CIRCUMSTANCES OF THE ECLIPSE.

	h m	Longitude from Greenwich. ° ′	Latitude. ° ′
Eclipse begins February 13	1 53.9	137 45.9 W.	58 32.5 S.
Central eclipse begins	3 38.8	117 0.7 E.	76 21.0 S.
Central eclipse ends	5 8.5	28 24.5 E.	41 4.5 S.
Eclipse ends	6 52.2	7 10.8 W.	10 32.9 S.

A Partial Eclipse of the Moon will occur on Feb. 28, 1896. It will not be visible in the Americas, but will be seen generally throughout Europe, Asia and Africa. At the middle of the eclipse 0.87 of the Moon's diameter will be immersed in the shadow.

ELEMENTS OF THE ECLIPSE.

Greenwich mean time of conjunction in right ascension, February 28, $8^h\ 15^m\ 40^s.4$.

Sun's right ascension	$22^h\ 45^m\ 20^s.08$	Hourly motion		$9^s.38$
Moon's right ascension	10 45 20 .08	Hourly motion		136 .15
Sun's declination	7° 54′ 21″ 3 S.	Hourly motion		0′ 56″.7 N.
Moon's declination	7 16 18 .4 N.	Hourly motion		17 30 .7 S.
Sun's equa. hor. par.	8 .9	Sun's true semidiam.		16 8 .5
Moon's equa. hor. par.	61 15 .5	Moon's true semidiam.		16 40 .7

TIMES OF THE PHASES.

Moon enters penumbra	February 28	5^h	$14^m.4$	Greenwich mean time.
Moon enters shadow		6	16 .3	
Middle of eclipse		7	45 .8	
Moon leaves shadow		9	15 .1	
Moon leaves penumbra		10	16 .7	

COMET NOTES.

Ephemeris of Comet *c* 1895 (Perrine).

The following ephemeris is constructed from Professor Campbell's elements as given elsewhere in this number.

1896.	Greenwich Mean Time. App. α		App. δ		Br.
	h	m	°	′	
January 21.5	19	36.2	− 9	20	0.65
23.5		37.2	− 8	39	
25.5		38.1	− 8	1	
27.5		39.0	− 7	23	
29.5		39.8	− 6	47	0.42
31.5		40.6	− 6	10	
February 2.5		41.4	− 5	35	
4.5		42.1	− 5	00	
6.5		42.8	− 4	26	0.29
8.5		43.4	− 3	52	
10.5		44.0	− 3	19	
12.5		44.6	− 2	45	
14.5		45.1	− 2	13	0.22
16.5		45.5	− 1	39	
18.5		45.9	− 1	7	
20.5		46.3	− 0	33	
22.5		46.5	− 0	2	0.18
24.5		46.7	+ 0	31	
26.5		46.9	+ 1	4	
28.5		47.0	+ 1	37	
30.5	19	47.0	+ 2	9	0.14

Mt. Hamilton, Cal., January 6, 1896. C. D. PERRINE.

Faye's Comet on Feb. 1 will be in R. A. $0^h\ 6^m$ and Decl. + 1° 16′.

Brook's Comet *d*, 1895 on the same date will be in R. A. $2^h\ 2^m$, Decl. + 63° 16′.

These are both too faint for any but the largest telescopes. We have no ephemerides beyond February 1.

For Perrine's comet see the article by the discoverer on another page.

The Orbit of Comet 1879 V.—In No. 362 of the *Astronomical Journal*, Dr. Kurt Laves of the University of Chicago gives the results of his computations on the definition orbit of Comet 1879, V, which was discovered by Palisa at Pola, Aug. 21, 1879. The final elements, depending upon 129 observations, indicate that the orbit does not differ sensibly from a parabola.

$$\begin{aligned} T &= 1879,\ \text{Oct.}\ 4.662208 \\ \pi &= 202^\circ\ 37'\ 21''.06 \\ \omega &= 115\ \ 26\ \ 23\ .22 \\ \Omega &= \ 87\ \ 10\ \ 57\ .84 \\ i &= \ 77\ \ \ 7\ \ 52\ .04 \\ \log q &= 9.9954633 \end{aligned}$$

GENERAL NOTES.

The Constant of Gravitation.—Professor William A. Rogers, of Colby University, Waterville, Me., has given elsewhere a full and very clear statement of his views concerning the meaning of k. The article is printed in small type because our space was already largely used when it came to hand. The small type does not mean that the article is of minor importance. On the contrary it is of great value to all interested in the study of this theme.

We have asked Professor Boys to furnish us good photographs of his apparatus that we may soon present the remainder of his excellent paper.

Graphic Construction of Eclipses—Professor Wm. F. Rigge, S. J. Georgetown College Observatory, Washington, D. C., has already prepared three excellent articles on the above theme, two of which have been printed. We were sorry that it was necessary to divide the last one published, for the cuts were presented at a disadvantage. This was not wholly our fault. Our engraving was so poor that part of it was rejected and that which was used was not as good as it ought to have been. We will serve our worthy correspondent better in the future.

Observation of Sun Spots.—Mr. C. S. Woodard, of Ypsilanti, Mich., recently sent us four neat drawings of sun spots made by the aid of a telescope of a theodolite, with object glass 1¼-inches aperture. The solar disc and spots are drawn to scale on very closely squared paper, and, at a glance, show relative positions, size, and motions of them from day to day, in a very satisfactory way. Such practice is very useful.

Schaeberle's new Photographic Reflector.—It was Prof. Schaeberle's intention to furnish some photographs of the planet Jupiter to accompany his article elsewhere printed in this number, but the illness and death of a near relative stopped his work for a while, and called him sadly to his old home in Michigan.

The Phenomena of Aratus.—One of our correspondents informs us that the translator of the Phenomena of Aratus is N. L. Frothingham and that the publishers are, Crosby, Nichols & Co., Boston.

The Sun, by Professor Young.—This book first published in 1881, has just appeared in a new and revised edition. Considerable portions of the old texts have been re-written and the revision has been thoroughly done, and where needed new parts and illustrations have been added. It is now up to date with a first rate article on helium. Fuller notice will be given later.

Mr. Lowell's Book on Mars.—It is a pleasant surprise to read Mr. Percival Lowell's new book on Mars. It contains a most readable account of what has been known of the planet up to last opposition and a full statement of his own work during the last two years. It is a very useful popular work that any one may read with very great profit. More will be said of this and Flammarion's recent work on the same subject in later issues.